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ABSTRACT

These materials are intended to provide meaningful mathematical experiences for pre-algebra students. These experiences emphasize the development of computational skills, mathematical concepts, and problem-solving techniques. This bulletin may be used as the basis for the first term of a one-year course. The complete course is divided into 12 basic units, six of which are presented here. Units one through six are: (1) formulas; (2) introduction to algebra; (3) metric measurement; (4) plane geometry; (5) money management; and (6) probability and statistics. Each individual lesson includes the aim, performance objectives, essential vocabulary, specific teaching suggestions, applications, and a summary. The mathematical skills and concepts are arranged in a sequential order allowing for the spiraling of skills among units and semesters. (PK)

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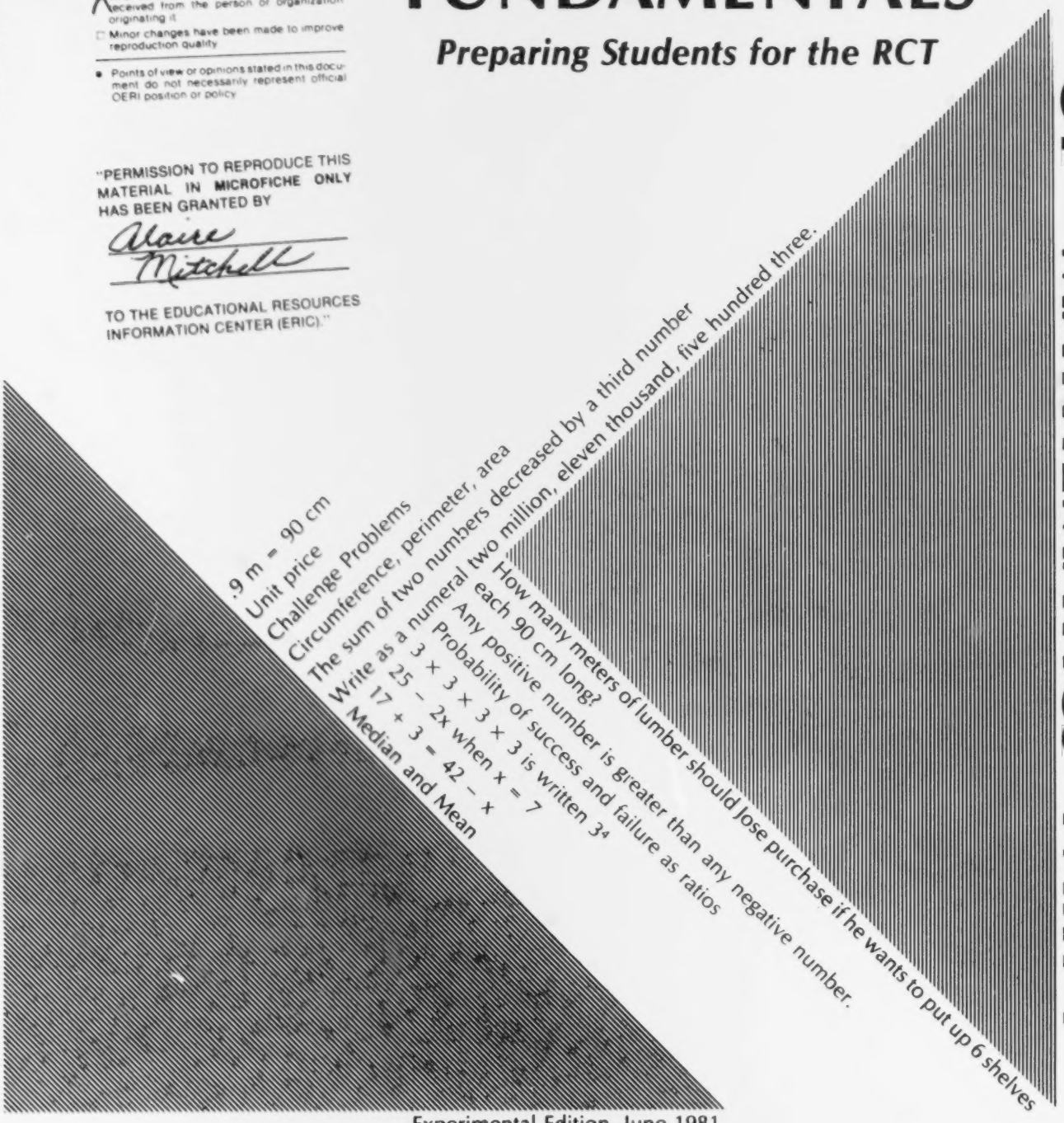
FUNDAMENTALS

Preparing Students for the RCT

OF MATHEMATICS PART I


How many meters of lumber should Jose purchase if he wants to put up 6 shelves each 90 cm long?
Any positive number is greater than any negative number.
Probability of success and failure as ratios
Write as a numeral two million, eleven thousand, five hundred three.
The sum of two numbers decreased by a third number
Circumference, perimeter, area
Challenge Problems
Unit price
.9 m = 90 cm
3 × 3 × 3 × 3 is written 3⁴
25 − 2x when x = 7
17 + 3 = 42 − x
Median and Mean

Experimental Edition, June 1981

NEW YORK CITY PUBLIC SCHOOLS
BOARD OF EDUCATION OF THE CITY OF NEW YORK
OFFICE OF CURRICULUM DEVELOPMENT AND SUPPORT • DIVISION OF CURRICULUM & INSTRUCTION

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Foreword

As part of an ongoing process of field testing and updating curriculum materials, this edition of Fundamentals of Mathematics--Part I is, in fact, a revised version of the earlier bulletin published in November, 1978. It is designed to provide meaningful mathematical experiences for pre-algebra students. These experiences emphasize the development of computational skills, mathematical concepts, and problem-solving techniques. For some students, this course will contribute to later succes in algebra; for others, it represents part of a program in everyday mathematical literacy.

This bulletin may be used as the basis for the first term of a one-year course, or for the first year of a two-year course. The complete course will be divided into twelve basic units, six of which are presented here. Each individual lesson includes:

aim
performance objectives
essential vocabulary
specific teaching suggestions
applications
summary

The mathematical skills and concepts are arranged in a sequential order allowing for the spiraling of skills among units and between semesters. Challenging and practical mathematical experiences are provided to stimulate student interest and to help overcome the many arithmetic deficiencies students may have. All is done in the belief that the development of mathematical concepts calls for careful and patient teaching throughout the year.

The units in this new edition are consistent with units of the New York State bulletin General High School Mathematics published August, 1978. The fist seven units of this New York State publication make up the curriculum on which the New York State Regents Competency examination is based. The units presented in this bulletin parallel those in General High School Mathematics, New York State Bulletin, August 1978. A cross-reference between the topics of the two sets of units is provided. There is a strong emphasis on those skills, concepts and solving techniques which are measured by the Regents Competency Test. To this end, the actural Regents Competency items are marked by asterisks.

Teachers and supervisors using the curriculum bulletin are urged to fill out the questionnaire at the back of the book and return it to the Office of Curriculum Development and Support by the appointed date.

CHARLOTTE FRANK
Executive Director

Acknowledgments

The curriculum bulletin, Fundamentals of Mathematics: Part I, was prepared by the Division of Curriculum and Instruction, Charlotte Frank, Executive Director.

Irwin Kaufman, Director of Mathematics, and Marvin Itzkowitz, Assistant to the Director of Mathematics, provided the overall supervision.

This publication is a revision of the experimental curriculum bulletin, Fundamentals of Mathematics Part I, published in 1978.

That manuscript was written by: Jacob Cohen, Assistant Principal/Mathematics, Theodore Roosevelt High School; Alice Farkouh, Assistant Principal/Mathematics, Prospect Heights High School; Edward Williams, Assistant Principal/Mathematics, Washington Irving High School.

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Special thanks is given to the following who made suggestions which are incorporated in this bulletin: Mary Gahn, Assistant Principal/Mathematics, John Bowne High School; Diana Radovnikovich, Mathematics Coordinator, Manhattan High School Superintendent's Office; Cecelia Cullen, Program Manager, High School Mathematics Skills; Eileen Petrozelli, Executive Assistant in the Office of the Queens High Schools Superintendent; Alice Farkouh, Assistant Principal/Mathematics, Prospect Heights High School.

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Unit I. Formulas

LESSON 1

AIM: To use symbols to represent numbers and operations

PERFORMANCE OBJECTIVES: The student will be able to

- state the operation associated with the words: sum, product, difference, quotient, more than, less than, diminished by, increased by, factor, etc.
- translate an English phrase expressing a binary operation into numerical symbols
- use a variable in a binary expression
- represent a product of two or more numbers in the following four ways: X, ·, ( ), and no sign at all.
- evaluate numerical expressions containing only one binary operation where the numbers are whole numbers

VOCABULARY:
New Terms: binary operation, factor, variable, evaluate

CHALLENGE PROBLEM:
On Monday 525 seniors were present in school. On Tuesday there were 484 seniors present. How many more seniors were present on Monday than on Tuesday?

DEVELOPMENT:

1. Discuss challenge problem. How did you arrive at an answer? Which key words told you which operation to use?
2. Elicit and list the four fundamental operations of arithmetic and their symbols: i.e. +, -, X, ÷, ( ), ·, ——, $\sqrt{}$. Since each of these operations is performed on two numbers, they are called binary operations. Relate binary to other familiar words whose prefix is bi such as bicycle and bicentennial.
3. Develop with the class a list of key words signaling binary operation Help students to develop an appreciation of the brevity, clarity, and convenience of a symbol of operation.

"How many different words can you think of which are used in place of add?" e.g. plus, sum, more than, increased by, exceeds.
Do the same for subtraction, multiplication, and division.

4. Do application A.

5. Introduce the meaning of variable as a symbol which represents "any number". Show that the product of 3 and "any number" may be written 3 X a, 3(a), 3·a or 3a where "a" is "any number."
Any letter may be used to indicate a variable. Define factor: if 2 or more numbers are multiplied, each is a factor of the product. Therefore, 2 and 5 are factors of 10, and 3 and a are factors of 3a, because they are multiplied together.

6. Do application B.

APPLICATIONS:

A.1. For each phrase, state the operation indicated.

2. Translate each phrase into symbols.

3. Evaluate each expression:

a.	nine plus three	9+3
b.	four increased by seven	4+7
c.	the sum of fifty-two and seventeen	52+17
d.	twenty-five more than eight	8+25
e.	one hundred three decreased by forty-seven	103-47
f.	the difference between seventy and thirty-four	70-34
g.	twenty-three less than two hundred	200-23
h.	the product of seven and thirteen	7·13
i.	the quotient of eighty-seven and three	87÷3
j.	twelve diminished by eleven	12-11
k.	from ten subtract eight	10-8

B.1. For each phrase state the operation indicated.

2. Use a variable to represent "a number" and write each phrase in algebraic symbols.

a.	the sum of 4 and a number	4+x
b.	a number decreased by 5	n-5
c.	16 divided by a number	16÷a

d. the product of 3 and a number	3x
e. a number increased by 13	k+13
f. twice a number	2b
g. 10 more than a number	n+10
h. 5 times a number	5a
i. 7 less than a number	x-7
j. 15 decreased by a number	15-k
k. from 23 subtract a number	23-n
l. from a number subtract 18	x-18

(Note to Teacher: Any letter may be used in each exercise.)

SUMMARY:

1. The factors of a number are 7 and 13. Find the number.
2. Express "the product of 7 and 13" in three different ways.

CLIFFHANGER:

What is the result of 801 diminished by a number, if the number is 357?

LESSON 2

AIM: To evaluate algebraic expressions with one or more variables

PERFORMANCE OBJECTIVES: The student will be able to:

- translate a phrase containing one or more variables into an algebraic expression.
- replace variables in an algebraic expression with their designated numerical values.
- change the format of a numerical expression for computational purposes.
- perform the indicated operations in numerical expressions.
- differentiate between algebraic and numerical expressions.

VOCABULARY:

New Terms: substitution, numerical value

Review: variable, binary operation, evaluate

CHALLENGE PROBLEM:

What is the result of 801 diminished by a number, if the number is 642?

DEVELOPMENT:

1. Discuss both the cliff-hanger and the challenge problems. Review the use of a variable in translating phrases into algebraic symbols. Express challenge problem as: 801 - x.
2. Elicit from the class the concept that variables are used to represent numbers. These variables can be given specific numerical values. Expressions containing variables can be evaluated once specific values are substituted for these variables. In doing the substitution the students should discover that algebraic phrases are written horizontally. Most calculations use other formats.
3. Return to the challenge problem and establish a pattern for evaluating an expression.

ALGEBRAIC EXPRESSION (TRANSLATION)	NUMERICAL EXPRESSION (SUBSTITUTION)	COMPUTATIONAL FORM
801 - x when x = 642	801 - 642	801 -642 159

Answer: 801- x = 159 when x = 642

4. Do application A. Emphasize the use of the format presented. The students should rewrite each expression showing the substitution and the computational form.

5. Write new expressions containing more than one variable.

   a. The sum of x, y, and z.

   b. the product of 5, a, and b Answers: x + y + z; 5ab

6. Have the students evaluate the expressions obtained in item 5 for the values shown below.

ALGEBRAIC EXPRESSION	NUMERICAL EXPRESSION	COMPUTATIONAL FORM
a) x + y + z when x = 257, y = 42, and z = 562	257 + 42 + 562	257 42 +562
b) 5ab when a = 9 and b = 12	5 · 9 · 12	5 × 9 = 45 45 × 12

Emphasize that the operations are binary and that when more than two numbers are used it may be necessary to write more than one computational form. Note: In (b) symbols have been used to indicate multiplication when numerals replace variables.

7. Do application B.

APPLICATIONS:

A. Evaluate each expression. Use the value of the variable indicated.

1. 4 + x; x = 12
2. n - 5; n = 51
3. $\frac{6175}{y}$; y = 19
4. 13p ; p = 0
5. x + 275; x = 493
6. 24(s) ; s = 15

B. 1. Represent each English phrase in algebraic symbols.

	Answers
a. the product of b and h	bh
b. twice the product of x and y	2xy
c. the sum of two numbers decreased by a third number	x+y-z
d. the number whose factors are seven, m, and n	7 mn

2. Name the operation or operations indicated in each expression:

a. bh c. $\frac{d}{r}$ e. 2L + 2W

b. p + i d. prt f. s - c

3. Evaluate each expression using the value of the variables shown in the table.

a	b	c	A	x	y
9	2	3	0	27	18

a. ac

b. c + x + y - A

c. b • b • b

d. abcA

e. $\frac{x}{c}$

f. y - b

SUMMARY:

Have students describe the steps necessary to evaluate an expression containing variables.

1. Write the expression in algebraic form.
   (An algebraic expression is a phrase containing variables.)
2. Substitute the appropriate numerical value for each variable.
   (Contrast algebraic expressions with numerical expressions: numerical expressions are phrases containing only numbers and operations.)
3. Compute - (All numerical expressions can be reduced to a single number through computations.)

LESSON 3

AIM: To evaluate expressions using the correct order of operations

PERFORMANCE OBJECTIVES: The student will be able to:

- state the order in which operations are to be performed
- evaluate numerical expressions using the correct order of operations
- evaluate algebraic expressions for stated values of the variables, using the correct order of operations
- state that a numerical expression has only one value

VOCABULARY:

New Terms: order of operations

Review: numerical value

CHALLENGE PROBLEM:

a. Write in numerical symbols: eight increased by two times five.

b. Find the result.

DEVELOPMENT:

1. Elicit the response to part a of the challenge problem: $8 + 2 \cdot 5$. Write the results obtained in response to part b on the chalkboard. In most cases, both 18 and 50 will be given. Ask the students to explain how they obtained their results. Elicit the fact that two answers were found due to the order in which the students performed the operations. Since everyone must get the same meaning from the expression $8 + 2 \cdot 5$, agreement must be reached on the order in which operations are to be performed.

   Inform pupils that when several operations are indicated in an expression, we proceed as follows:

   a. First perform multiplications and divisions, going in order from left to right.
   b. Then perform the additions and subtractions, going in order from left to right.

2. Demonstrate the order of operations in the following problems:

   a. The challenge: $8 + 2 \cdot 5 = ?$

      i. Perform multiplications and divisions, going from left to right. $2 \cdot 5 =$

$$\begin{array}{r} 2 \\ \times\ 5 \\ \hline 10 \end{array}$$

      ii. Then perform additions and subtractions, going from left to right. $8 + 10 =$

$$\begin{array}{r} 8 \\ +\ 10 \\ \hline 18 \end{array}$$

$8 + 2 \cdot 5 = ?$
$8 + 10 = ?$
18 answer

   b. The challenge: $24 \div 2 \times 3 = ?$

      i. Perform multiplications and divisions, going from left to right.

$24 \div 2 \times 3 = ?$
$12 \times 3 = ?$
36 answer

   c. The challenge: $24 \times 2 - 18 \div 3 + 15 = ?$

      i. Perform multiplications and divisions, going from left to right.

$24 \times 2 - 18 \div 3 + 15 = ?$
$48 - 6 + 15 = ?$

      ii. Then perform additions and subtractions, going from left to right.

$48 - 6 + 15 = ?$
$42 + 15 = ?$
57 answer

3. Do application A.

4. Present the class with these algebraic expressions -- to evaluate.

   a. $25 - 2x$, when $x = 7$ b. $563 - 21p$, when $p = 17$

Have the students use the format that was established in a previous lesson. As you proceed through the first example, make the students aware that, once the substitution has been made, the resulting numerical expression must be evaluated, using the order of operations.

ALGEBRAIC EXPRESSION (translation)	NUMERICAL EXPRESSION (substitution)	COMPUTATIONAL FORM	
$25 - 2x$	$25 - 2 \cdot 7 = 11$	$\begin{array}{r} 2 \\ \times\ 7 \\ \hline 14 \end{array}$	$\begin{array}{r} 25 \\ -14 \\ \hline 11 \end{array}$
$563 - 21p$	$563 - 21 \cdot 17 = 206$	$\begin{array}{r} 21 \\ \times\ 17 \\ \hline 357 \end{array}$	$\begin{array}{r} 563 \\ -357 \\ \hline 206 \end{array}$

5. Review the order of operations and do application B.

APPLICATIONS:

A. Find the value of each expression:

1. $8 + 9 - 4$ (13)
2. $48 \div 3 \times 5$ (80)
3. $10 + 5 \cdot 21$ (115)
4. $167 - 9 \cdot 7$ (104)
5. $18 \times 4 \div 2$ (36)
6. $16 + 10 \times 2 - 8$ (28)
7. $127 - 93 \div 3 + 11$ (107)

B. Evaluate each of the following expressions for the indicated value of x.

1. $3x + 2 \; ; \; x = 7$ (23)
2. $116 - 9x; \; x = 12$ (8)
3. $\frac{7x}{2} \; ; \; x = 18$ (63)
4. $16x + 9 \; ; \; x = 0$ (9)
5. $23x - 92; \; x = 8$ (92)
6. $\frac{9x}{4} + 3 \; ; \; x = 12$ (30)
7. $2x + 5 + 3x; \; x = 4$ (25)
8. $241 - 22x \; ; \; x = 7$ (87)
9. $\frac{10053}{x} \; ; \; x = 9$ (1117)
10. $14x + x - 34; \; x = 4$ (26)

SUMMARY:

1. Why was it necessary to establish an order of operations?
2. State the order of operations for addition, subtraction, multiplication and division.

CLIFF-HANGER:

Joseph earns $8 in salary plus $2 in tips each afternoon he works. Last week he worked 5 afternoons. How much did he earn?

LESSON 4

AIM: To use parentheses as a grouping symbol

PERFORMANCE OBJECTIVES: The student will be able to:

- recognize that operations within parentheses are to be performed first.
- evaluate numerical expressions with parentheses.
- recognize the "distributive property of multiplication over addition"

VOCABULARY:

New Terms: parentheses, distributive property of multiplication over addition

Review: order of operations

CHALLENGE PROBLEM:

Joseph earns $8 in salary plus $2 in tips each afternoon he works. Last week he worked 5 afternoons. How much did he earn? Write a numerical expression showing how your answer was obtained.

DEVELOPMENT:

1. Discuss the challenge problem. Elicit from the class that Joseph earned a total of $10 each arternoon. The students should attempt to rewrite this problem using numbers and symbols. Have students realize that the result of $8 + 2 \times 5$ does not satisfy the problem, because $8 + 2 \times 5$ means 18.
2. Elicit a need to group numbers when addition or subtraction must be done first. Teach the symbol ( ), parentheses, as a "do me first"- i.e. state that operations within ( ) are to be computed first. The proper representation of the challenge problem is $(8 + 2) \cdot 5$.
3. Amend the order of operations to read:
   a. simplify expressions within the parentheses.
   b. perform all multiplications and divisions from left to right.
   c. perform additions and subtractions from left to right.
4. Do application A.
5. Go back to challenge problem. Consider doing the problem a different way. What is the total salary paid to Joseph in one week? (8) 5 = 40 What is the total amount Joseph received in tips? (2) 5 = 10 How much did he earn? 40+10=50 Write this in symbols: (8) 5 + (2) 5=50

Compare this with the original solution (8 + 2) 5 = 50

This means that (8)5 + (2) 5 = (8+2) 5, and demonstrates the distributive property of multiplication over addition.

6. Show that 3 (4 + 2) and 3 (4) + 3 (2) are equivalent:

$$3\ (4 + 2) \quad = ? \quad 3\ (4) + 3\ (2)$$
$$3\ (6) \ = ? \quad 12 + 6$$
$$18 \qquad\quad 18$$

Thus, 3 (4+2) = 3 (4) + 3 (2) also demonstrates the distributive property.

7. Do application B.

APPLICATIONS:

A. Perform the indicated operations

1. 8 (9 - 4)	(40)	5. (35 - 10) ÷ 5 + 2	(7)
2. 8 9 - 4	(68)	6. 35 - 10 ÷ 5 + 2	(35)
3. 48 ÷ 3 + 5	(21)	7. 18 - 2·6	(6)
4. 48 ÷ (3 +5)	(6)	8. (18 - 2) 6	(96)

B. Which expression in column B is equivalent to an expression in column A?

A	B
1. 2 (4 + 5)	a. 19(21) + 19(19)
2. 5 (7 + 3)	b. 2(4) + 2 (5)
3. (7 + 3) 9	c. 300
4. 12 (8 + 17)	d. 5 (10)
5. 19 (40)	e. (7) 9 + (3) 9
	f. 2(4) + 5

SUMMARY:

Write a numerical expression to indicate that the sum of 6 and 9 is to be multiplied by 3.

LESSON 5

AIM: To evaluate numerical expressions containing exponents

PERFORMANCE OBJECTIVES: The student will be able to:

- express a number written in exponential form as a repeated multiplication.
- state the meaning of base, exponent and factor.
- evaluate numerical expressions containing exponents.
- compute the squares and cubes of numbers.

VOCABULARY:

New Terms: base, exponent, square, cube, power

Review: factor

CHALLENGE PROBLEM:

Find the result of using 3 as a factor 4 times.

DEVELOPMENT:

1. Review the meaning of "factor" as implying multiplication.

   Have the class agree that the challenge problem means $3 \cdot 3 \cdot 3 \cdot 3$ which equals 81.

2. Review the advantage of using shorthand notation, and elicit the the advantage of a shorthand notation for repeated multiplication. State that $3 \cdot 3 \cdot 3 \cdot 3$ is written as 3^4. (This is read as "3 to the fourth power.") Emphasize the relative positions of the numerals in this expression and the need for neatness on the part of the students.

3. Define the terms base and exponent, using 3^4 as an example.

   The base (3) is the number used as a factor. The exponent (4) is the number of times the base is used as a factor.

4. Do applications A and B.

5, Introduce the term "square" to represent an exponent of 2 and "cube" to represent an exponent of 3.

6. Present the expression 3×10^2 for evaluation. Students should be aided in rewriting all expressions containing exponents in multiplication (factor) form. It must be emphasized that the exponent refers only to its base and to no other number. Therefore, 3×10^2 means

$3 \times 10 \times 10 = 300$

Provide practice in several problems such as:

$5 \times 10^3 = 5 \times 10 \times 10 \times 10$

$6 \cdot 7^2 = 6 \cdot 7 \cdot 7$

7. Do applications C and D.

APPLICATIONS:

A. Express each of the following using exponents:

1. $2 \times 2 \times 2 \times 2$
2. $10 \times 10 \times 10$
3. $1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1$
4. $3 \cdot 3 \cdot 4 \cdot 4 \cdot 4$
5. 10 (Note: $10 = 10^1$)
6. $11 \cdot 11$

B. Name the base and exponent in each expression:

1. 10^4 2. 4^{10} 3. 2^6 4. 1^8 5. 15^1

C. Rewrite each expression in multiplication (factor) form and evaluate:

1. 2^5 2. 10^2 3. $2^3 \cdot 4^2$ 4. $5^2 + 2^2$

D. Find the square and the cube of each number:

1. 3 2. 1 3. 15 4. 47

SUMMARY:

1. Evaluate $2^2 + 3^2$ and $(2 + 3)^2$.
2. Have the students name the bases and the exponents in each expression.
3. Compare the values of each of the expressions and have the students recognize that the two expressions are not equivalent.

LESSON 6

AIM: To evaluate algebraic expressions containing exponents

PERFORMANCE OBJECTIVES: The student will be able to:

- evaluate algebraic expressions containing exponents.
- rewrite variables with exponents as repeated multiplications.

VOCABULARY:

Review: exponent, base, factor, substitution, numerical value

CHALLENGE PROBLEM:

1. Write $6a^4$ as an expression in multiplication (factor) form.
2. Find its value when a=2.

DEVELOPMENT:

1. Write $6a^4$ as $6 \cdot a \cdot a \cdot a \cdot a$
2. Review the meaning of exponent, base and factor.
3. Review substitution of numerical values in algebraic expressions.
4. Evaluate $6a^4$ when a = 3 as follows:

ALGEBRAIC EXPRESSION	NUMERICAL EXPRESSION (SUBSTITUTION)	COMPUTATIONAL FORM
$6a^4$ when a = 3	$6 \cdot 3 \cdot 3 \cdot 3 \cdot 3$	Left to student

5. Do application A.
6. Demonstrate a procedure for evaluating algebraic expressions with more than one variable.

   Find the value of gt^2 when g = 16 and t = 5

ALGEBRAIC EXPRESSION	NUMERICAL EXPRESSION	COMPUTATIONAL FORM
gt^2, g=16, t=5	$16 \cdot 5 \cdot 5$	$\begin{array}{r} 16 \\ \times 5 \\ \hline 80 \end{array}$ $\begin{array}{r} 80 \\ \times 5 \\ \hline 400 \end{array}$

Answer: gt^2=400 when g=16 and t=5

7. Do application B.

APPLICATIONS:

A. Given a = 5, b = 3 find the value of:

1. a^2
2. $3a^3$
3. $2a + 3a^2$
4. $\frac{2b^2}{3}$
5. $b + b^2$

B. Using the values x = 2, y = 3, z = 4, p = 10 and q = 12, evaluate each of these expressions:

1. $\frac{pq}{z}$
2. $\frac{pq^2}{2}$
3. $x + y^2 + z^3$
4. $5p^2$
5. $x^2y - 12$
6. $xyz + 13$
7. $p + pqx$
8. $2(y + z)$

SUMMARY:

Have students list the steps in evaluating an algebraic expression containing exponents.

CLIFF-HANGER:

I heard on the radio that the temperature was going to be between 25° Celsius and 30° Celsius. Is that warm enough to go swimming?

LESSON 7

AIM: To evaluate the subject of formulas

PERFORMANCE OBJECTIVES: The student will be able to:

- state a need for using formulas
- state the difference between a formula and an expression
- recognize the subject of a formula
- find the value of the subject of a formula

VOCABULARY:

New Terms: formula, subject of a formula, mathematical sentence, perimeter

Review: Evaluate, substitution

CHALLENGE PROBLEM:

On a recent trip to Canada, I heard on the radio that the temperature would be between $25^{o}C$ and $30^{o}C$. Was it warm enough to swim that day?

DEVELOPMENT:

1. Discuss challenge problem. Elicit the need for a conversion formula from Celsius to Fahrenheit.

   Give the formula to the students and have them compute the two temperatures:

   $F = \frac{9C}{5} + 32$ when $C = 25$ and when $C = 30$

   When $C = 25$ $F = \frac{9 \times 25}{5} + 32$; When $C = 30$ $F = \frac{9 \times 30}{5} + 32$

   $25 \times 9 = 225$ $\quad 5\overline{)225} = 45$ $\quad 45 + 32 = 77$

   $30 \times 9 = 270$ $\quad 5\overline{)270} = 54$ $\quad 54 + 32 = 86$

$F = 77^{o}$ $F = 86^{o}$

Was it warm enough to swim?

2. Have the students recognize that in order to solve this problem a formula was needed. Where do formulas come from? Use the following to demonstrate one way in which a formula can be obtained.

Sam had to find the perimeters of various squares. He placed his information in this form. Where S was the length of a side of the square and P was the perimeter.

S	P
3	12
5	20
6	24
10	40
12	?

a. How can you find the value of P when S is 12?

b. Find the missing perimeter.

c. How would you get the result if "S" were any other number?

Elicit the formula $P = 4S$. Stress that P represents the number of units in the perimeter and S represents the number of units in the length of the side. Find P if $S = 23$.

3. Have students recall other formulas they know from past experience. Some formulas might be:

$P = 2l + 2w$ $V = lwh$

$A = lw$ $V = e^3$

$A = s^2$ $i = prt$

$A = \frac{1}{2}bh$ $d = 16t^2$

$d = rt$ $L = A + (N - 1)d$

What will each formula help you find or determine? Tell students that the quantity that you can find using a formula is called the subject of the formula. Have students state the subject of each formula already written.

4. State that a formula is a mathematical sentence which contains a subject, verb (=), and phrase.

5. Practice evaluating the subjects of formulas. Do application A. Select formulas with a wide variety of numerical substitutions. In each case emphasize the importance of careful organization of work and stress the rules for the order of operations. (Begin with formulas from students' past experiences.)

APPLICATION:

Find the value of the subject of each formula when the other variables are known.

1. $A = lw$ $l = 8$ $w = 8$
2. $P = 2 + 2$ $l = 7$ $w = 8$
3. $V = lwh$ $l = 5,\ w = 6$ $h = 7$
4. $d = 16t^2$ $t = 3$
5. $L = A + (N - 1)d$ $A = 17,\ N = 12,\ d = 2$

SUMMARY:

Why are formulas useful?

UNIT I: FORMULAS

SUGGESTED TEST ITEMS

1. On the line to the right of each phrase write the symbol of operation ($+, -, \times, \div$) that is indicated by the phrase. Do NOT compute.

   a. the difference between x and y _____

   b. pq _____

   c. the quotient of 625 and 15 _____

   d. 397 increased by Q _____

   e. $\frac{D}{t}$ _____

   f. the product of 35 and 710 _____

   g. five less than 25 _____

2. Rewrite each English phrase in numerals and symbols.

   a. 27 divided by the number n.

   b. 16 more than x.

   c. the product of 7, s, and t

   d. the sum of a, b, and c

   e. from 34 subtract nine

3. Find the value of each numerical expression.

   a. $10 + 3 \cdot 2$

   b. $12 \div 3 \times 2$

   c. $16 + 9 (3) - 16$

   d. $(10 + 4) \cdot 3$

   e. $42 \div (3 + 4)$

   * f. 6^2

   * g. 4×10^3

4. Use the table of values for the variables to evaluate each algebraic expression.

a	b	c	x	y
5	7	2	18	30

a. $2b + 3$

b. $c + x + y$

c. $13a$

d. $\frac{x}{c}$

e. $x + b - c$

f. $y (a - 3)$

g. c^4

h. $2a^2$

5. Find the value of the subject of each formula.

a. $P = 2 (L + W)$ when $L = 23$ and $W = 19$

b. $A = bh$ when $b = 12$ and $h = 9$

c. $p = 2s + b$ when $s = 6$, and $b = 8$

d. $V = e^3$ when $e = 7$

* 6. The formula $F = \frac{9C}{5} + 32$ is used to calculate the Fahrenheit temperature when the Celsius temperature is known. Find the Fahrenheit temperature when the Celsius temperature is 20^o.

7. On the line to the left of each phrase in column I, write the letter corresponding to the item in column II which matches that phrase.

Column I	Column II
_____ the square of a number	a. the 4 in 3^4
_____ a variable	b. the 6 in 6^7
_____ an exponent	c. x^2
_____ factors	d. x^3
_____ base	e. the b in b + 9
_____ an example of the	f. 12 in x – 12
distributive property of	g. 5 (2 + 7) = 5 • 2 + 7
multiplication over	h. 5 (2 + 7) = 5 (2) + 5 (7)
addition	i. the numbers 4 and x in 4x.

8. Add: 463
38
246

9. Multiply: 5,432
x 22

*10. From 892 subtract 574.

*11. Divide: 32) 6,848

Unit II. Introduction to Algebra

LESSON I

AIM: To use symbols of equality and inequality

PERFORMANCE OBJECTIVES: The student will be able to

- compare two whole numbers verbally using the words "is greater than","is less than," "or is equal to."
- use symbols of equality and inequality to compare two whole numbers.
- recognize whether a sentence using $<$, $=$, or $>$ is true or false.

VOCABULARY:

New Terms: equality, inequality, is greater than, is less than

Review: mathematical sentence, statement

CHALLENGE PROBLEM:

Make each of the following statements true by using one of the expressions: is equal to, is greater than, or is less than.

1. $12 + 5$ _____ $1 + 4 \times 4$ (is equal to)
2. $6 \div 3 \times 2$_____ 1 (is greater than)
3. $2 \times 3 + 7$_____ $2 \times (3 + 7)$ (is less than)

DEVELOPMENT:

1. After the challenge problem is completed, the teacher reviews the equality symbol and introduces the symbols of inequalities. Students write the appropriate symbol in place of the words in each of the challenge problems:

$12 + 5 = 1 + 4 \times 4$

$6 \div 3 \times 2 > 1$

$2 \times 3 + 7 < 2 \times (3 + 7)$

2. Have the students record this information in their notebooks.

= is equal to

> is greater than

< is less than

3. Have students do applications.

APPLICATIONS:

A. Comparing numbers

By placing a "T" or "F" following each statement, indicate which are true and which are false.

1. $7 < 21$ _____
2. $33 > 26$ _____
3. $15 < 13$ _____
4. $12 > 16$ _____
5. $73 < 69 + 2$ _____
6. $17 + 3 < 42 - 20$ _____
7. $63 < 39 + 24$ _____
8. $47 < 53 - 5$ _____
9. $14 > 19 + 10$ _____
10. $27 < 83 - 64$ _____

B. Write the symbol of equality or inequality which makes each statement true.

1. $7 \_\_\_ 14$
2. $10 \_\_\_ 9$
3. $20 + 10 \_\_\_ 30$
4. $72 \_\_\_ 15 + 61$
5. $2^3 \_\_\_ 3^2$
6. $2\ (3+5) \_\_\_ 2(3) + 2\ (5)$
7. $8 \cdot 3 + 2 \_\_\_ 40$
8. $2 \times 10^2 \_\_\_ 40$
9. $7\ (15 - 7) \_\_\_ 56$
10. $10 \div 5 \times 2 \_\_\_ \frac{25}{5}$

C. If $x = 5$ use the symbol $<$, $>$, or $=$ to make each sentence true.

1. $x + 3 \_\_\_ 7$
2. $5x \_\_\_ 30$
3. $2x + 3 \_\_\_ 13$
4. $16 - x \_\_\_ 10$

SUMMARY:

Review reading, writing, and interpreting equality and inequality symbols.

LESSON 2

AIM: To order numbers on a number line

PERFORMANCE OBJECTIVES: The student will be able to

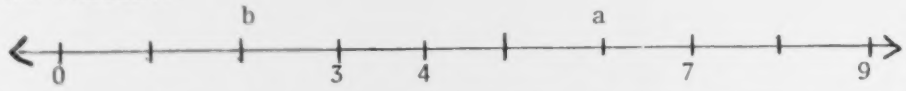
- construct a number line using a unit length with end points at 0 and 1, and extending to include all whole numbers.
- locate any positive whole number on the number line; thereby, showing understanding of the one-to-one correspondence between whole numbers and points on the line.
- compare the positions of two points on the line to their relative size (smaller is to the left).
- describe the order of two numbers on the number line using $<$ and $>$.
- describe the order of two lettered points on the line using $<$ and $>$.
- state the rule, "if a is to the right of b, then $a > b$ or $b < a$.
- understand the correspondence of rational numbers to points on the number line by subdividing a unit on the number line (quarter units only).

VOCABULARY:

New Terms: number line, correspondence, fraction, rational number, mixed number, ordering, unit.

Review: is less than, is greater than, is equal to

CHALLENGE PROBLEM:



Use the symbol $<$, = or $>$ to make the statement true: a____b

Explain how you arrived at your answer.

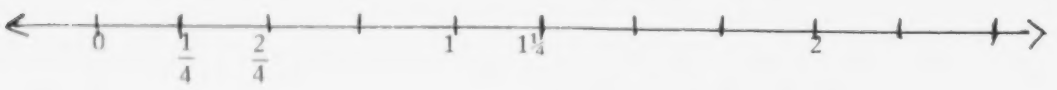
DEVELOPMENT:

Discuss the one-to-one correspondence between real numbers and points on the line. Establish the ordering property for positive numbers and zero, eliciting the property that any number to the

right of another number is the greater of the two. State the rule as: "If a is to the right of b, then $a > b$ or $b < a$."

2. Do application A
   Do application B 1 - 5

3. Add to the number line, so that some rational numbers are included. Discuss the method of dividing the whole to include rational numbers. Confine discussion to quarters only.



Review meaning of fraction as: $\frac{\text{number of parts taken}}{\text{number of parts in the whole}}$

Define rational numbers as all whole numbers and fractions.

Have the students locate the rational numbers 3, $\frac{3}{4}$, and $1\frac{3}{4}$ on the number line.

Discuss the differences between a whole number, a proper fraction, and a mixed number, all of which are rational numbers.

Review the rule: "If a is to the right of b then $a > b$."

Apply this rule to the ordering of rational numbers.

4. Do Applications B, 6-10 and C.

APPLICATIONS:

A. What number is;

1. 2 units to the right of 4?
2. 3 units to the left of 6?
3. 4 units to the right of 0?
4. 5 units to the right of 7?
5. 7 units to the right of 3?
6. 5 units to the left of 5?
7. 4 units to the left of 5?
8. 6 units to the left of 9?

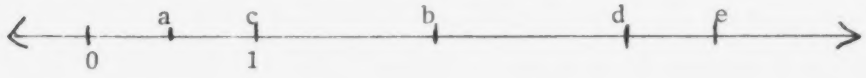
9. 7 units to the left of 7?

10. 0 units to the right of 9?

B. For each pair of numbers name the number which will appear further to the right on number line.

1. 4, 9
2. 15, 3
3. 724, 953
4. 0, 5
5. 351, 350
6. 3, $2\frac{1}{2}$
7. $1\frac{1}{4}$, $\frac{3}{4}$
8. $\frac{1}{4}$, $\frac{3}{4}$
9. $7\frac{2}{4}$, $7\frac{1}{4}$
10. $8\frac{1}{4}$, $6\frac{1}{4}$

C. Using the number line, write the symbols $<$, $=$, or $>$ in each space to make the statement true.



1. a_____c
2. e_____d
3. 1_____b
4. d_____0
5. 1_____c
6. b_____a

SUMMARY:

Use the following equally spaced points on a number line to answer the questions below.

1. If a is given the value 1, what is the value of each of the other letters?
2. If c is given the value 1, what is the value of each of the other letters?
3. Of the letters c, a, f, and e, which represents the largest number and which represents the smallest?

LESSON 3

AIM: To introduce ruler fractions

PERFORMANCE OBJECTIVES: The student will be able to

- recognize when two rational numbers are equivalent (denominators 2,4,8,16).
- use or to describe the order of ruler fractions.
- change an improper fraction into a mixed number and vice versa.

VOCABULARY:

New Terms: equivalent fractions, improper fractions, mixed numbers, numerator, denominator.

CHALLENGE PROBLEM:

Given two number line pictures:

1)

Name the fraction located at point a.

2)

Name the fraction located at point b.

Express the relationship ($<$, $>$, =) between these 2 fractions.

DEVELOPMENT:

1. Discuss challenge problem. Elicit that the fractions at a and b ($\frac{2}{4}$ and $\frac{4}{8}$) are equivalent. Discuss other ways to name $\frac{1}{2}$ such as $\frac{3}{6}$, etc. Name 1 in various ways (i.e. $1 = \frac{4}{4} = \frac{8}{8}$). Locate $\frac{1}{4}$ on line 1. Find an equivalent fraction on line 2.

2. Discuss the markings on a standard ruler where each inch is divided into 16 equal parts. Have students construct a number line where the space between 0 and 1 is divided into 16 equal parts. Find equivalent names for $\frac{3}{8}$ and $\frac{3}{4}$.

   Elicit rule: To find an equivalent form of a fraction, multiply the numerator and denominator by the same number.

3. Do application A.

9. Do application E.

APPLICATIONS:

A. Rewrite each fraction in an equivalent form:

1. $\frac{7}{8} = \frac{\quad}{16}$
2. $\frac{3}{4} = \frac{\quad}{16}$
3. $\frac{1}{4} = \frac{\quad}{8}$
4. $\frac{2}{4} = \frac{\quad}{16}$

B. Answer T (true) or F (false):

1. $\frac{5}{10} > \frac{6}{10}$
2. $\frac{1}{3} < \frac{2}{3}$
3. $\frac{1}{4} < \frac{1}{8}$
4. $\frac{3}{4} < \frac{3}{8}$
5. $\frac{1}{2} > \frac{7}{16}$
6. $\frac{7}{8} > \frac{13}{16}$

C. Write the correct symbol ($=, >, <$)

1. $\frac{3}{4} \bigcirc \frac{7}{8}$
2. $\frac{5}{8} \bigcirc \frac{1}{2}$
3. $\frac{4}{8} \bigcirc \frac{1}{2}$
4. $\frac{5}{16} \bigcirc \frac{3}{4}$

D. Write each mixed number as an improper fraction:

1. $1\frac{1}{4} =$
2. $3\frac{3}{8} =$
3. $4\frac{1}{2} =$
4. $2\frac{7}{16} =$

E. Write each improper fraction as a mixed number:

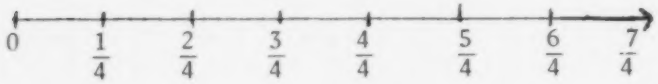
1. $\frac{7}{2} =$
2. $\frac{13}{4} =$
3. $\frac{13}{8} =$
4. $\frac{16}{5} =$

SUMMARY:

Review the rule, for changing mixed numbers to improper fractions and improper fractions to mixed numbers. Illustrate with at least two applications.

4. Locate $\frac{5}{8}$ and $\frac{1}{2}$ on the number line. Which is larger (by being further to the right)? Have students write $\frac{1}{2}$ as $\frac{4}{8}$ and compare. Thus, $\frac{5}{8} > \frac{4}{8}$ because it is further to the right. Therefore, $\frac{5}{8} > \frac{1}{2}$. Elicit that, in ordering fractions, if the denominators are the same then the numerators can be compared.

5. Do applications B and C.

6. 

The points on the line are shown in terms of fourths.

Have students recall that $\frac{4}{4}$ equals 1. Recall that the point, $\frac{3}{4}$ units to the right of 1, was written $1\frac{3}{4}$. Thus $\frac{7}{4}$ names the same point as $1\frac{3}{4}$ and $\frac{7}{4} = 1\frac{3}{4}$.

To change from a <u>mixed number</u> to an <u>improper fraction</u> proceed as follows:

Method A: $1\frac{3}{4} = 1 + \frac{3}{4}$

$1\frac{3}{4} = \frac{4}{4} + \frac{3}{4}$

or $1\frac{3}{4} = \frac{7}{4}$

Method B: $1\frac{3}{4} = \frac{4 \times 1 + 3}{4}$

$1\frac{3}{4} = \frac{4+3}{4}$

$1\frac{3}{4} = \frac{7}{4}$

7. Do application D.

8. To change from an improper fraction to a mixed number, proceed as follows:

Method A: $\frac{9}{4} = \frac{4}{4} + \frac{4}{4} + \frac{1}{4}$

$\frac{9}{4} = 1 + 1 + \frac{1}{4}$

$\frac{9}{4} = 2\frac{1}{4}$

or

Method B: $\frac{9}{4} = 4\overline{)9}\ \begin{matrix} 2 \\ \\ 8 \\ \hline 1 \end{matrix} = 2\frac{1}{4}$

LESSON 4

AIM: To read a ruler

PERFORMANCE OBJECTIVES: The student will be able to:

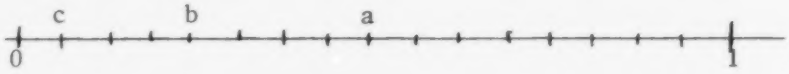
- read the markings on a ruler.
- measure line segments with a ruler.
- name line segments using two capital letters.
- compare lengths of line segments.

VOCABULARY:

New Terms: ruler, line segment, end point, lowest terms

Review: unit, equivalent fractions

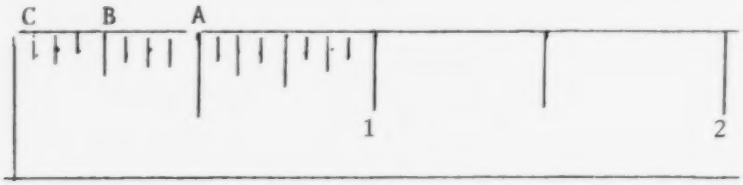
CHALLENGE PROBLEM:



Name the fractional number on the number line associated with each letter. (c = $\frac{1}{16}$ b = $\frac{4}{16}$ or $\frac{2}{8}$ or $\frac{1}{4}$ a = $\frac{8}{16}$ or $\frac{4}{8}$ or $\frac{2}{4}$ or $\frac{1}{2}$)

DEVELOPMENT

1. Discuss challenge problem. Elicit possible equivalent answers for letters a, b, and c. (Define lowest terms.)
2. Compare the letters on the number line with the letters marked on a ruler (16^{th} ").



3. Provide each student with a ruler and discuss its markings.
4. Do application A.
5. Develop the procedure for measuring a line segment. Emphasize the necessity for having one end point of the line segment at the zero point of the ruler.

6. Indicate different notations for naming a line segment.

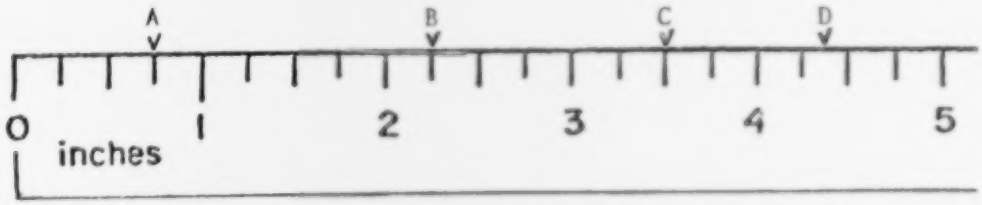
   (i.e. ____a____, A_____B is $\overline{AB}$)

7. Do application B .

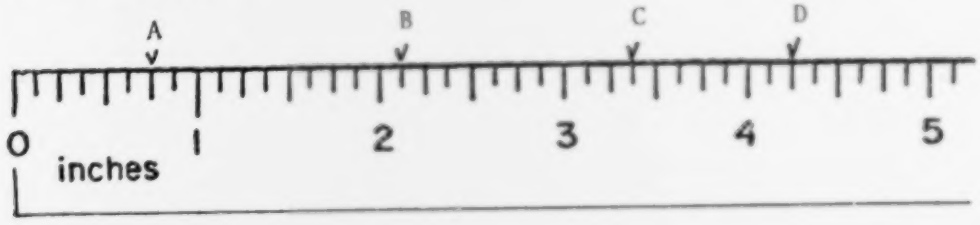
8. Compare lengths of line segments in application C.

APPLICATIONS: The following applications should be reproduced on a rexograph and distributed to students.

A.



1. Read the values of A, B, and C. Discuss D. This should lead students to understand the need for dividing the inch into smaller units.



2. Read the values of A, B, C, and D. (Use the ruler.)

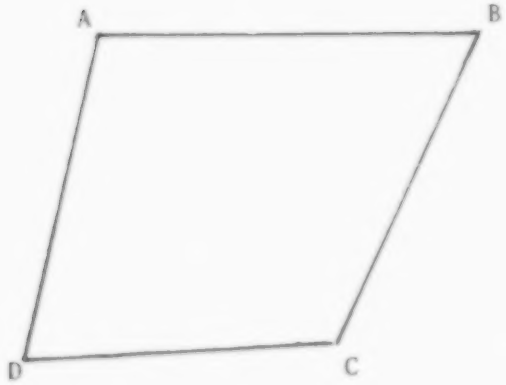
B. Name each line segment and find its length.

A _____ B C _____ D

_____ a _____ b

(Note: Express fractions in lowest terms.)

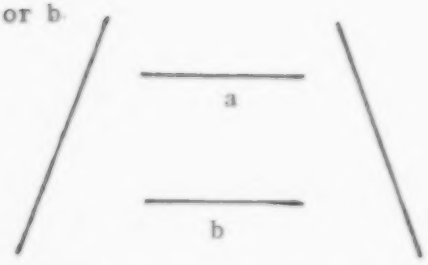
C. Using a ruler, find the measure of each side shown. Arrange your findings by segment name, in size order, from smallest to largest.



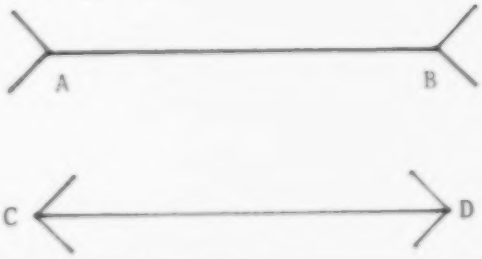
Segment	Measure

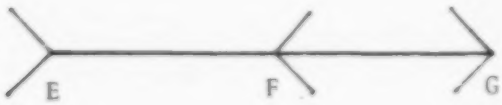
D. In each of the pairs of segments shown below, without using a ruler, select the shorter segment. Circle your answer.

a or b

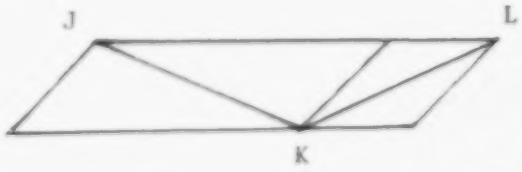


$\overline{AB}$ or $\overline{CD}$





$\overline{EF}$ or $\overline{FG}$



$\overline{JK}$ or $\overline{KL}$

LESSON 5

AIM: To comprehend the meaning of directed numbers

PERFORMANCE OBJECTIVES: The student will be able to:

- represent gains and losses, right and left, above and below zero, etc. as signed numbers.
- write the opposite of a given situation.
- write the opposite of a given signed number.

VOCABULARY:

New Terms: signed numbers, positive, negative, opposites, withdrawal, deposit

CHALLENGE PROBLEM:

If the temperature on a winter day is 3^{o} and then drops 5^{o}, what is the new temperature?

DEVELOPMENT:

1. Discuss challenge problem. How do we write 2^{o} below zero? Define this as -2, read "negative 2," and 3^{o} above zero as +3, read "positive 3." Explain that a number without a sign is positive. Do application A.
2. Ask for other life situations in which there is a need for negative numbers. Elicit such examples as profit and loss, north and south, east and west, above and below sea level, rise and fall, deposit and withdrawal, gain and loss of yardage in football.
3. Do applications B and C.
4. Elicit that a gain and loss of the same amount are called opposites.
5. Do application D.

APPLICATIONS:

A. Read each of the following and state whether the number is above or below zero.

1. -4	3. -13
2. +8	4. 5

B. Express each change described in this table in words and by a signed number:

Article	OLD PRICE	NEW PRICE	CHANGE
1 liter gasoline	27¢	33¢	up 6¢ or + 6¢
1 lb. margarine	93¢	89¢	
1 lb. bread	96¢	99¢	

C. Represent each of the following situations as a signed number:

1. jumping 4 feet forward
2. withdrawing $10 from the bank
3. losing $3\frac{1}{2}$ pounds in weight
4. a stock price falling $4\frac{5}{8}$ points
5. earning $7
6. a debt of $25
7. the cost of coffee dropping 43¢ a pound
8. walking down 12 steps

D. Describe the opposite of each of these situations:

1. earning $15
2. a 7^{o} rise in temperature of
3. a 20 yard gain
4. a deposit of $25
5. a profit of $10

SUMMARY:

Have the students list the new vocabulary words learned today and explain their meanings.

LESSON 6

AIM: To extend the number line to include negative numbers

PERFORMANCE OBJECTIVES: The student will be able to:

- draw the number line to the left of 0 to allow for movement in both directions.
- rename the points on the number line using positive and negative numbers.
- define movement to the right on a number line as positive and movement to the left as negative.
- express the distance between 2 points as a signed number.

VOCABULARY:

New Terms: integer

Review: opposite, one-to-one correspondence

CHALLENGE PROBLEM:

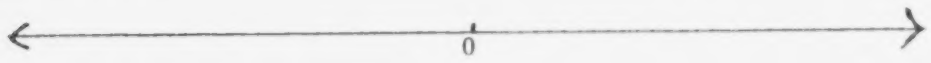
On a thermometer scale, a reading of 10^{o} above zero is marked by a point located 10 units above the zero mark. Where is the mark located that indicates 10^{o} below zero?

DEVELOPMENT:

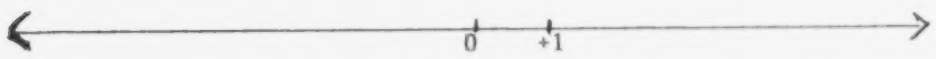
1. Refer to the challenge problem.
   Elicit that the mark indicating 10^{o} below zero is 10 units below the zero mark. Thus, temperatures of 10^{o} above zero and 10^{o} below zero are represented by marks which are the same distance from the zero mark, but on opposite sides of it.
2. Which integer could be used to represent 10^{o} above zero? (+10) or 10^{o} below zero? or (-10).
   Have pupils recall that +10 is opposite of -10.
3. Which set of integers would you associate with the whole-number readings which are:
   a. Above zero on a thermometer scale (set of positive integers)
   b. Below zero (set of negative integers)?
4. Help pupils to realize that a thermometer scale actually represents a number line in a vertical position. A number line in a horizontal position has positive integers paired with points to the right of zero, negative integers paired with points to the left of zero.

5. Have pupils associate the integers with points on the number line as follows:

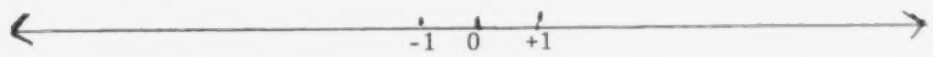
   a. Draw a picture of a line in horizontal position and mark a point. Label it zero.



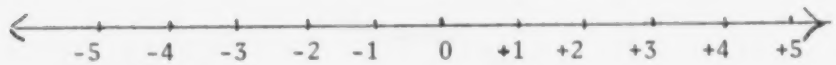
   b. Choose a unit length and mark off one unit length to the right of zero. Assign +1 to this point.



   c. Mark off one unit length to the left of zero (the same distance but opposite in direction). Since +1 is the opposite of -1, we will assign -1 to this point.



   d. In a similar way, have pupils associate other pairs of opposites, such as +2 and -2, +3 and -3, etc. with points on the number line.



   Emphasize that in a pair of opposites each number is the same distance from 0, but on opposite sides of it.

   e. Pose questions such as:

   (1) With what number do we associate a point 5 units to the right of 0? What number is associated with a point 5 units to the left of 0?

(2) Where is the point that is paired with -8 located?
Where is the point that is paired with +6?

(3) Is there a point which is the opposite of zero?

6. Every integer may be paired with a point on the number line, but not every point on the number line corresponds to an integer.

Review that there is a one-to-one correspondence between the set of integers and some of the points on the number line.

7. Do application A.
8. Define movement to the right on a number line as positive and movement to the left as negative.
9. Do application B.
10. Pose the problem:

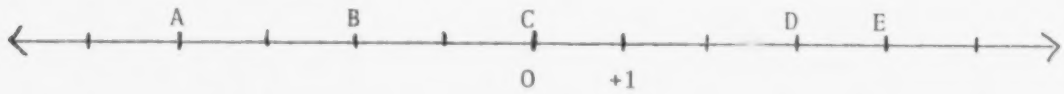
Using the number line, represent the change in temperature from $+3^{o}$ to -2^{o} as a signed number.

(Note to teacher: Do not discuss the operation of subtraction. Do this problem using change on the number line.)

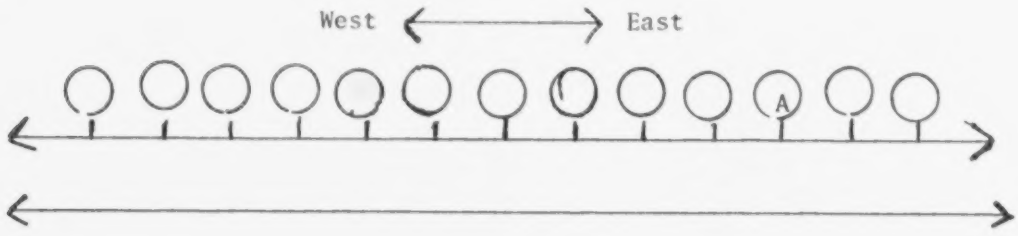
11. Do application C.

APPLICATIONS:

A. Write the signed number associated with each letter



B. In a trip taken along an East-West road, the trips taken to the East are positive and those taken to the West are negative. Each of the following gives the starting and ending points and the real number which measures the trip. Place the letter in each circle. Read the hidden message.



1. The trip from A to V is -4
2. The trip from S to V is -2
3. S to O is -3
4. O to T is +6
5. M to A is +1
6. M to E is -2
7. U to V is +5
8. V to L is -2
9. L to R is -4
10. V to H is +6
11. H to Y is -9
12. M to D is -7

C. Using a number line express each change as a signed number.

1. from +1 to +3
2. from +3 to +1
3. from -5 to -2
4. from -2 to -5
5. from -2 to +4
6. from +4 to -2

SUMMARY:

A large parking garage has 8 levels above ground level and 5 levels below. An elevator starting at ground level makes the following stops:

A: up 2 levels B: up 1 level C: down 4 levels

D: down 2 levels E: up 7 levels

1. Represent each of the points A, B, C, D, E as points on a vertical number line.

2. Write each distance as a signed number.
   a) From A to E
   b) From B to D
   c) From C to A
   d) From E to D

LESSON 7

AIM: To establish an ordering property of positive and negative rational numbers

PERFORMANCE OBJECTIVES: The student will be able to:

- express the order of two positive integers in terms of their position on the number line (a is to the left of b, so $a < b$).
- use the ordering symbols for all signed numbers (i.e., $-2 > -6$ because -2 is to the right of -6).
- find the least common denominator, given two or more fractions where the denominator of one is a multiple of the denominator of the other.

VOCABULARY:

New Terms: least common denominator (LCD)

CHALLENGE PROBLEM:

On Tuesday the thermometer showed a temperature of 6 degrees below zero (-6 degrees). On Wednesday, a temperature of 2 degrees below zero (-2 degrees) was recorded. Which is the higher of these two temperatures?

DEVELOPMENT:

1. Since the ordering property for positive rational numbers has already been established, the teacher is now able to use the number line to order negative numbers.
2. Elicit and list the following:
   a. Any positive number is greater than any negative number.
   b. All positive numbers are greater than 0.
   c. All negative numbers are less than 0.

Summarize the ordering relationships as follows:

If a is to the right of b, then $a > b$.

If a is to the left of b, then $a < b$.

If a and b name the same point, then $a = b$.

With these statements in view, the students should locate the two points -6 and -2 on the number line. They should see that -2 is to the right of -6 and therefore $-2 > -6$. Since this seems contradictory to students' intuitive feelings, many applications should be provided to insure understanding.

2. Do application A.
3. Locate $-\frac{1}{2}$ and $-\frac{3}{4}$. Which is larger? At this point teacher should review equivalent fractions. Introduce term <u>least common denominator</u> as the smallest denominator which has each of the other denominators as a <u>factor</u>. Elicit method for multiple families:<u>divide</u> largest denominator by each of the other denominators. If the result of each division is a whole number, the largest denominator is the least common denominator.
4. Do applications B and C.

APPLICATIONS:

A. Which is the higher temperature?

1. $+3^{o}$, $+5^{o}$
2. $+3^{o}$, -4^{o}
3. $+3^{o}$, -1^{o}
4. -4^{o}, $+5^{o}$
5. -3^{o}, -7^{o}
6. -4^{o}, -3^{o}
7. -12^{o}, $+21^{o}$
8. -1^{o}, -2^{o}
9. $+4^{o}$, -4^{o}
10. -5^{o}, -3^{o}
11. 0^{o}, -4^{o}
12. $+4^{o}$, 0^{o}

B. Which is the larger:

1. $+\frac{3}{8}$, $+\frac{1}{8}$
2. $-\frac{1}{4}$, $-\frac{2}{4}$
3. $-\frac{3}{5}$, $+\frac{1}{2}$
4. $+\frac{3}{4}$, $-\frac{1}{16}$
5. $+\frac{3}{5}$, $+\frac{1}{5}$
6. $-\frac{3}{8}$, $-\frac{1}{8}$

Note to teacher: Any positive number is greater than any negative number.

C. 1. State the least common denominator for each group of fractions below.

2. Change the fractions in each group to equivalent fractions with <u>that</u> least common denominator.

3. Arrange the fractions in each group in order from smallest to largest.

a) $+\frac{1}{2}, \quad +\frac{5}{8}$

b) $-\frac{9}{16}, \quad -\frac{3}{8}$

c) $+\frac{2}{9}, \quad +\frac{1}{3}, \quad +\frac{2}{27}$

d) $-\frac{1}{5}, \quad -\frac{3}{10}, \quad -\frac{3}{20}$

e) $+\frac{5}{12}, \quad +\frac{11}{24}, \quad -\frac{1}{2}$

SUMMARY: Have students state the ordering property for all signed numbers and the steps required to order rational numbers.

CLIFF-HANGER: Which is larger $-\frac{2}{3}$ or $-\frac{3}{4}$?

LESSON 8

AIM: To compare two rational numbers

PERFORMANCE OBJECTIVES: The student will be able to:

- find the least common denominator, given two fractions with relatively prime denominators.
- find the least common denominator, given two fractions with unlike denominators which are not relatively prime.
- use $<, >$, and = to compare rational numbers with unlike denominators.

VOCABULARY:

Review: least common denominator, unlike denominators

CHALLENGE PROBLEM:

Which is larger $-\frac{2}{3}$ or $-\frac{3}{4}$?

DEVELOPMENT:

1. Discuss challenge problem. Elicit from class that the method used in the previous lesson cannot be used because 3 is not a factor of 4. Lead the students to discover that the L.C.D. must have 3 and 4 as factors.

2. Determine that the LCD is 12, since 3 and 4 are its factors.
   Change $-\frac{2}{3}$ to $-\frac{8}{12}$ and $-\frac{3}{4}$ to $-\frac{9}{12}$

   Plot each point on the number line and compare their relative positions.

   Thus $-\frac{8}{12} > -\frac{9}{12}$ so $-\frac{2}{3} > -\frac{3}{4}$

   Practice additional problems before assigning applications such as:

   a) Which is smaller $+\frac{6}{7}$ or $+\frac{2}{3}$?

   b) Show that $-\frac{9}{15}$ is equivalent to $-\frac{3}{5}$.

3. Do applications A and B .

4. Have class compare $\frac{7}{10}$ and $\frac{13}{15}$. Elicit need for a second method to be used in finding the least common denominator. To find the least common denominator, take the largest denominator (which is 15) and multiply by 2, 3, 4, etc., until you find a number which has the smaller denominator (which is 10) as a factor.

Ex: 10
15: 15, 30, 45 . . . . .

Since $30 \div 10 = 3$, 30 is the least common denominator

$\frac{7}{10} = \frac{21}{30}$ $\quad \frac{13}{15} = \frac{26}{30}$ $\quad \frac{26}{30} > \frac{21}{30}$ so $\frac{13}{15} > \frac{7}{10}$

5. Do application C.

Applications:

A. Answer T (true) or F (false)

1. $+\frac{1}{4} < +\frac{1}{8}$
2. $+\frac{4}{5} < +\frac{5}{8}$
3. $+\frac{3}{4} < +\frac{3}{8}$
4. $-\frac{4}{8} > -\frac{4}{7}$
5. $+\frac{2}{3} < +\frac{5}{6}$
6. $+\frac{3}{5} > +\frac{4}{7}$
7. $-\frac{7}{8} > -\frac{13}{16}$
8. $-\frac{3}{6} = -\frac{4}{12}$
9. $-\frac{1}{2} > -\frac{7}{16}$
10. $+\frac{5}{12} > +\frac{1}{2}$

B. In the circle provided, write the correct symbol (=, >, <).

1. $-\frac{1}{4}$ ○ $\frac{1}{3}$

2. $\frac{1}{2}$ ○ $\frac{7}{12}$

3. $\frac{5}{6}$ ○ $\frac{11}{12}$

4. $-\frac{2}{3}$ ○ $-\frac{3}{4}$

5. $\frac{1}{12}$ ○ 0

6. $\frac{5}{12}$ ○ $\frac{1}{2}$

7. $\frac{3}{8}$ ○ $\frac{1}{3}$

8. $\frac{1}{4}$ ○ $\frac{1}{12}$

9. $\frac{14}{21}$ ○ $-\frac{2}{3}$

10. $\frac{3}{5}$ ○ $\frac{1}{2}$

C. In the circle provided, write the correct symbol (>, <, =)

1. $\frac{5}{6}$ ○ $\frac{3}{4}$

2. $\frac{5}{6}$ ○ $\frac{5}{8}$

3. $\frac{4}{15}$ ○ $\frac{3}{10}$

4. $\frac{5}{6}$ ○ $\frac{7}{9}$

5. $\frac{9}{4}$ ○ $\frac{13}{6}$

6. $1\frac{3}{4}$ ○ $1\frac{7}{10}$

SUMMARY:

Review the method for comparing two rational numbers.

LESSON 9

AIM: To compare more than two rational numbers

PERFORMANCE OBJECTIVES: The student will be able to:

- find the LCD for more than 2 fractions with unlike denominators
- order more than 2 rational numbers with unlike denominators

VOCABULARY:

Review: least common denominator (LCD), unlike denominators

CHALLENGE PROBLEM:

Which fraction has the largest value?

$$\frac{2}{3} \quad \frac{3}{5} \quad \frac{4}{9}$$

DEVELOPMENT:

1. Discuss challenge problem. Find the least common denominator and change all fractions to equivalent form. Use the method demonstrated previously.

2. Do applications A and B.

3. Arramge in order from smallest to largest $\frac{1}{3}, \frac{1}{2}, \frac{1}{7}, \frac{1}{5}$.

   Elicit that $\frac{1}{7}$ is the smallest because the whole is divided into more parts. Therefore, each part is smaller. Thus, $\frac{1}{7}, \frac{1}{5}, \frac{1}{3}$ and $\frac{1}{2}$ is the solution.

   Have the class arrange the numbers $\frac{2}{5}, \frac{2}{9}, \frac{2}{7}, \frac{2}{11}$ in order from smallest to largest.

   Students should be led to determine that it was not necessary to find a common denominator because the numerator was the same for all fractions in the group. A rule for ordering fractions that have the same numerator can now be stated by the students.

4. Do application C.

APPLICATIONS:

A. Find the LCD for each group of fractions

1. $\frac{1}{2}$, $\frac{3}{4}$, $\frac{7}{8}$

2. $-\frac{1}{3}$, $-\frac{5}{12}$, $-\frac{3}{8}$

3. $\frac{1}{5}$, $\frac{7}{8}$, $\frac{3}{10}$

4. $-\frac{1}{3}$, $-\frac{2}{5}$, $-\frac{1}{2}$

5. $\frac{5}{6}$ $\frac{11}{16}$ $\frac{7}{12}$

B. Change each fraction in A to an equivalent form with the least common denominator, and arrange each set in order from smallest to largest.

C. Which fraction is the largest ?

a) $\frac{2}{3}$, $\frac{2}{5}$, $\frac{2}{7}$, $\frac{2}{9}$

b) $\frac{1}{5}$, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$

SUMMARY:

Which fractions have a value between $\frac{1}{2}$ and $\frac{3}{4}$?

a) $\frac{5}{8}$ b) $\frac{11}{12}$ c) $\frac{2}{3}$ d) $\frac{4}{5}$

LESSON 10

AIM: To add signed numbers using the number line

PERFORMANCE OBJECTIVES: The student will be able to:

- add two whole numbers on a number line.
- add positive and negative numbers on the number line. (Sums may be either positive or negative.)
- state and demonstrate the commutative property of addition on the number line.

VOCABULARY:

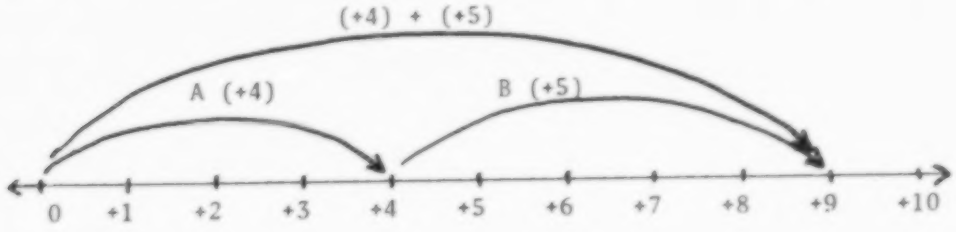
New Terms: commutative property

CHALLENGE PROBLEM:

The temperature at 8:00 a.m. was 4^{o} above zero. During the next two hours there was a gain of 5^{o}. What was the temperature at 10:00 a.m?

DEVELOPMENT:

1. Discuss challenge problem. Explain that the phrase "a gain of 5" from the point + 4 is equivalent to adding +5 to +4 or (+4) + (+5). (Parentheses are used to separate the signed number from the symbol of operation.)
2. Have the students draw a number line from 0 to 10 to find the sum of +4 and +5.



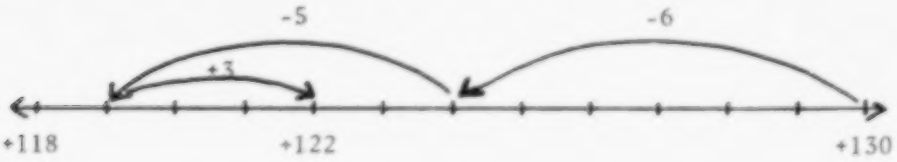
Arc A begins at 0 and moves 4 spaces to the right. It ends at +4.
Arc B begins at +4 and moves 5 spaces to the right. It ends at +9.
Thus: (+4)+(+5)= +9. The commutative property of addition should be stated and illustrated (+5) + (+4) = (+4) + (+5)

3. Do applications A and B.

4. Maribel weighed 130 lbs. in January. During February she lost 6 lbs. In March she lost 5 lbs., but in April she gained 3 lbs. What is her weight at the end of April?

   The students should be able to arrive at the weight of 122 lbs. The teacher should point out that the values could be written as the sum of the numbers: (+ 130) + (-6) + (-5) + (+3) =?

   The find the sum of these numbers, the number line can be employed with the starting point at +130.



   Thus +130 - 6 - 5 + 3 = +122

5. Do application C.

APPLICATIONS:

A. Illustrate the sum on each number line below.

1. (+7) + (+2)

2. (+5) + (-3)

3. 4 + 6

Note: Explain that 4 + 6 is another way of writing (+4) + (+6) and 9 - 6 is another way of writing (+9) + (-6)

4. 9 - 6

B. Use the number line to add the following:

+9	+6	*-3	*-9	+4	-5
-8	+4	-9	-3	-11	+5

*Illustrate commutative property

C. Find each of the following sums, using the number line if necessary.

1. (-5) + (-2)
2. (+5) + (+2)
3. (+5) + (-2)
4* (-8) + (+3)
5* (+3) + (-8)
6. (-4) + (-3) + (-7)
7. (-8) + (-3) + (+2) + (-1)
8. (+7) + (-9) + (-3) + (+10)

SUMMARY:

1. What is the result of adding +6 and +2?
   Will the sum of two positive numbers always be positive? Explain.
2. What is the result of adding -6 and -2?
   Will the sum of the two negative numbers always be negative? Explain.

CLIFF-HANGER:

What is the sum of +6 and -2? When a positive number and a negative number are added, how do we determine the sign of the sum?

LESSON 11

AIM: To develop a rule for the addition of signed numbers

PERFORMANCE OBJECTIVES: The student will be able to:

- state and apply the rules for adding signed numbers.
- define absolute value of an integer as its distance from 0 on a number line.
- define additive inverses as two numbers whose sum is 0.
- find the additive inverse of a number.

VOCABULARY:

New Terms: absolute value, additive inverse

CHALLENGE PROBLEM:

John had a balance of $275 in his checking account. He wrote a check for $317. Express this problem as the sum of two signed numbers and find his checking account balance as a signed number.
(Note to teacher: assume overdraft privileges.)

DEVELOPMENT:

1. Discuss challenge and cliff-hanger problems as examples of addition of signed numbers.
   Challenge: (+275) + (-317) = ?
   Cliff-hanger: (+2) + (-6) = ?
   Establish a need for a rule for adding signed numbers when the number line is not convenient (as in challenge).

2. Recall summary of lesson 10. Will the sum of two positive numbers always be positive? Will the sum of two negative numbers always be negative? Introduce term and symbols for absolute value of a number and define it as the distance of the number from zero on a number line. (Have students state the absolute values of several signed numbers.) Elicit the rule for addition of signed numbers with like signs by giving several examples until students discover: "Add the absolute values of the numbers and place the common sign in front of the sum."

3. Do applications A and B.

4. Return to challenge problem to see if the rule applies. Continue in the same manner to elicit the rule for addition of signed numbers with unlike signs. Students should discover, from examples given, the rule: "find the difference between the absolute values, and place the sign of the number that has the greater absolute value in front of the difference."

5. Do application C.

6. Have students note that in problems 7 and 8 we added opposites, and that the sum of two opposites is zero. Define additive inverses (or opposites) as two numbers whose sum is zero.

APPLICATIONS;

A. Find the value of each of the following:

1. $|+3|$
2. $|-7|$
3. $|9+15|$
4. $|6| + |-2|$
5. $|+12| + |-3| - |+1|$

B. Find the sum for each of the following:

1. $\begin{array}{r} +10 \\ +\ 7 \\ \hline \end{array}$
2. $\begin{array}{r} +29 \\ +13 \\ \hline \end{array}$
3. $\begin{array}{r} -41 \\ -34 \\ \hline \end{array}$
4. $(-12) + (-14)$
5. $(-17) + (-56) + (-42)$
6. $(+932) + (+1034) + (+32)$

C. Find each sum:

1. $\begin{array}{r} +25 \\ -10 \\ \hline \end{array}$
2. $\begin{array}{r} -18 \\ +24 \\ \hline \end{array}$

Note: Students may use the commutative property to change to the more convenient form.

$$\begin{array}{r} +24 \\ -18 \\ \hline \end{array}$$

3. (-48) +(+36) =

4. (-107) + (+79) =

5. (-9) + (-12) +(+30) =

6. (-27) + (+63) + (-41) =

7. (-8) + (+8) =

8. (+237) + (-237) =

SUMMARY:

1. Have students state each rule for adding signed numbers.

2. Have students compute the following:

   (+19) + (-48) + (+48) + (-19)

   Discuss possible methods of calculating including use of additive inverses.

3. What is the absolute value of (-3)?

4. What is the additive inverse of (-3)?

LESSON 12

AIM: To add rational numbers with like signs

PERFORMANCE OBJECTIVES: The student will be able to:

- add rational numbers with like denominators and like signs.
- add rational numbers with unlike denominators and like signs

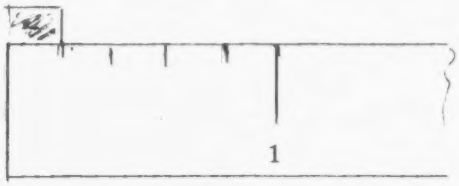
VOCABULARY:

Review: mixed numbers, like denominators, unlike denominators, lowest terms, signed numbers

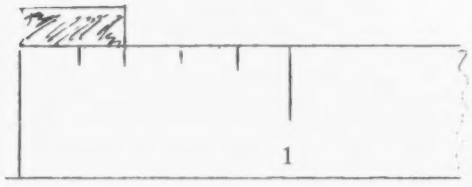
CHALLENGE PROBLEM:

Mr. Jones measured two pieces of steel using the ruler shown below. What is the length of steel bar A? What is the length of steel bar B? What is the combined length of both bars?

BAR A:



BAR B:



DEVELOPMENT:

1. Discuss challenge problem. Represent these fractions as positive numbers and illustrate their sum on a number line. Elicit a rule for adding fractions with like denominators. Extend discussion to include addition of mixed numbers where fractions have like denominators (ex: $(-3\frac{1}{4}) + (-2\frac{3}{4})$).
2. Do application A.
3. Consider the problem $(+\frac{2}{3}) + (+\frac{1}{5})$.
   Discuss the differences between this example and the challenge.

Elicit from the students that to add fractions the denominators must be the same. In work on comparing fractions, methods were discussed for obtaining a least common denominator. Use one of these methods to change $(+\frac{2}{3})$ and $(+\frac{1}{5})$ to fractions with like denominators. Extend discussion to include addition of mixed numbers where fractions have unlike denominators.

4. Do application B.

APPLICATIONS:

A. Apply the rules for adding signed numbers and adding fractions with like denominators. Express results as proper fractions or mixed numbers in lowest terms.

1. $(+\frac{1}{4}) + (+\frac{3}{4})$
2. $(-\frac{3}{10}) + (-\frac{4}{10})$
3. $(+\frac{5}{12}) + (+\frac{11}{12}) + (+\frac{1}{12})$
4. $(-\frac{7}{8}) + (-\frac{5}{8}) + (-\frac{3}{8})$
5. $(+4\frac{5}{6}) + (+3\frac{5}{6})$
6. $(-8\frac{1}{5}) + (-3\frac{2}{5})$
7. $(+\frac{2}{3}) + (+5\frac{1}{3}) + (+6\frac{2}{3})$

B. Find the sums.

1. $(+\frac{2}{3}) + (+\frac{5}{6})$
2. $(+\frac{3}{4}) + (+\frac{1}{12})$
3. $(+\frac{7}{8}) + (+\frac{1}{5})$
4. $(-\frac{3}{4}) + (-\frac{5}{6})$
5. $(+14\frac{1}{6}) + (+12\frac{2}{3})$
6. $(-4\frac{1}{3}) + (-2\frac{5}{8}) + (-3\frac{1}{4})$

SUMMARY:

Have students state the rules for adding:

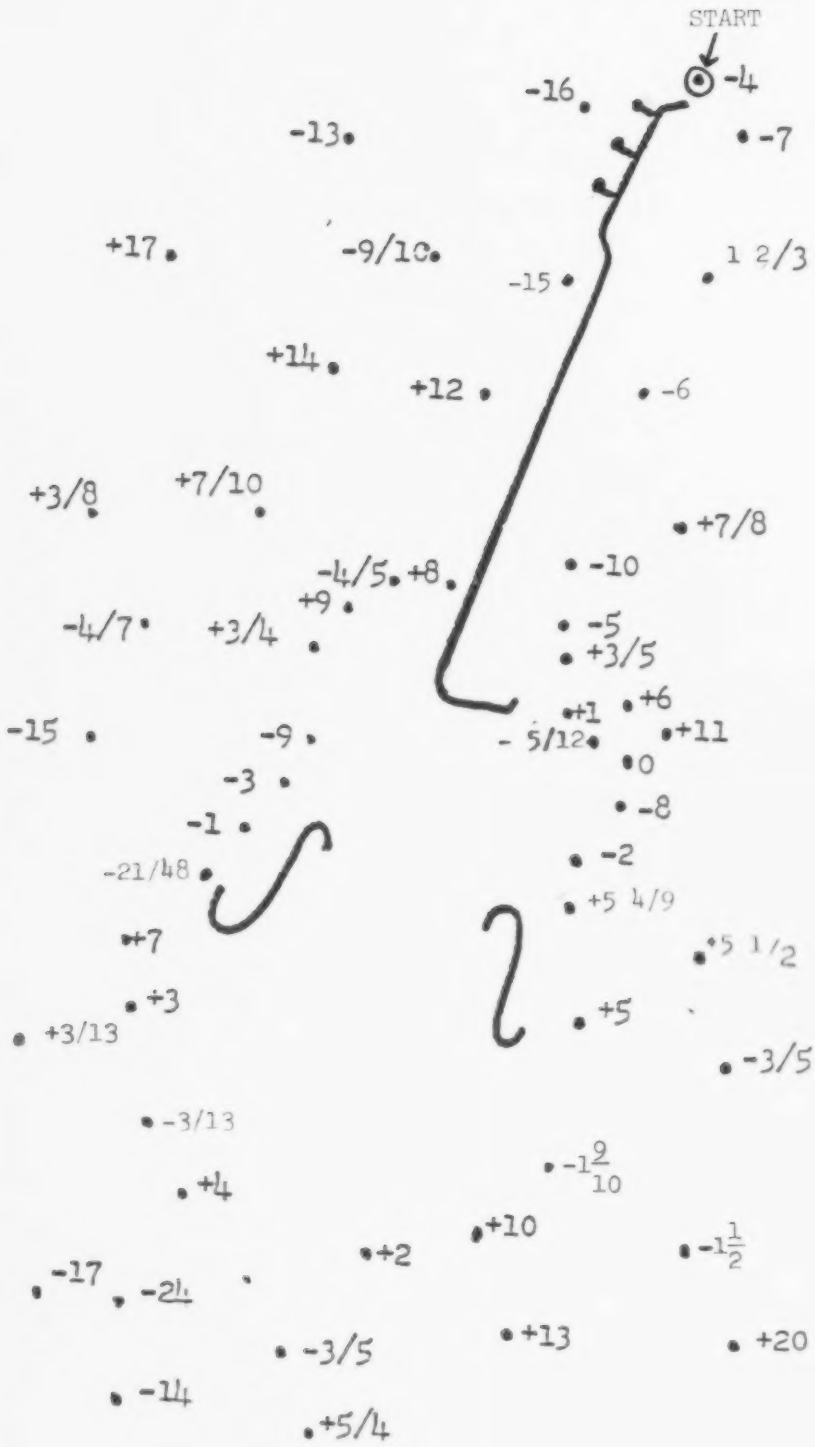
1. fractions with like denominators
2. fractions with unlike denominators
3. signed numbers with like signs

SUGGESTED HOMEWORK

Solve the problems below. Then locate the answers on the Dot-o-gram on the opposite page. Follow the answers in order to make a picture. For example, the answer to Number 1 is -4; so you start at -4.

1. $(-3) + (-1) = -4$
2. $(0) + (-7) =$ ___
3. $(+\frac{1}{3}) + (+\frac{4}{3}) =$ ___
4. $(-3) + (-3) =$ ___
5. $(-1) + (-3) + (-6) =$ ___
6. $(-3) + (-2) =$ ___
7. $(+\frac{1}{5}) + (+\frac{2}{5}) =$ ___
8. $(+5) + (-4) =$ ___
9. $(-\frac{1}{6}) + (-\frac{1}{4}) =$ ___
10. $(-1) + (+9) + (-2) =$ ___
11. $(-2) + (+2) =$ ___
12. $(-1) + (-3) + (-4) =$ ___
13. $(-4) + (+2) =$ ___
14. $(+2\frac{1}{9}) + (+3\frac{1}{3}) =$ ___
15. $(+4) + (+1) =$ ___
16. $(-\frac{2}{5}) + (-\frac{3}{2}) =$ ___
17. $(+20) + (-10) =$ ___
18. $(+3) + (-1) =$ ___
19. $(+3) + (+1) =$ ___
20. $(-\frac{2}{13}) + (-\frac{1}{13}) =$ ___
21. $(+4) + (-1) =$ ___
22. $(+5) + (-1) + (+3) =$ ___
23. $(-\frac{3}{6}) + (-\frac{1}{16}) =$ ___
24. $(-2) + (+1) =$ ___
25. $(-7) + (+2) + (+2) =$ ___
26. $(-10) + (+1) =$ ___
27. $(+\frac{1}{4}) + (+\frac{1}{2}) =$ ___
28. $(+8) + (-1) + (+2) =$ ___
29. $(-\frac{3}{5}) + (-\frac{1}{5}) =$ ___
30. $(+4) + (+5) + (-1) =$ ___
31. $(-3\frac{5}{7}) + (-2\frac{6}{7}) + (-8\frac{3}{7}) =$ ___

DOT - O - GRAM


START
-4
-16
-13
-7
+17
-9/10
1 2/3
-15
+14
+12
-6
+3/8
+7/10
+7/8
-4/5
+8
-10
+9
-4/7
+3/4
-5
+3/5
+6
+1
-15
-9
- 5/12
+11
0
-3
-8
-1
-2
-21/48
+5 4/9
+7
+5 1/2
+3
+3/13
+5
-3/5
-3/13
-1 9/10
+4
+10
+2
-1 1/2
-17
-24
+13
+20
-3/5
-14
+5/4

LESSON 13

AIM: To add rational numbers with unlike signs

PERFORMANCE OBJECTIVES: The student will be able to:

- add rational numbers with like denominators with unlike signs
- add rational numbers with unlike denominators with unlike signs
- subtract rational numbers

VOCABULARY:

New Terms: exchange (regroup)

Review: least common denominator (L.C.D.)

CHALLENGE PROBLEM:

On a number line, represent the sum $(+\frac{7}{8}) + (-\frac{6}{8})$.

DEVELOPMENT:

1. Discuss the challenge problem. Review the rule for adding signed numbers with unlike signs. Elicit a rule for adding fractions with like denominators and unlike signs.
2. Do application A.
3. Consider the problem $(+\frac{3}{4}) + (-\frac{1}{5})$.
   Find the least common denominator and apply the rule for adding fractions with unlike signs.
4. Do application B.
5. Part of the rule for adding fractions with unlike signs is to find the difference between the absolute values of the fractions. Therefore, have students practice subtraction of mixed numbers Present the concept of <u>exchange</u> (regrouping).
6. Do application C.

APPLICATIONS:

A. Find the sums of the following fractions:

1. $(+\frac{5}{6}) + (-\frac{2}{6})$
2. $(-\frac{3}{7}) + (+\frac{8}{7})$
3. $(-\frac{17}{9}) + (+\frac{4}{9})$
4. $(+\frac{1}{11}) \quad (-\frac{3}{11})$

B. Find the sums of the following fractions:

1. $(-\frac{2}{3}) + (+\frac{4}{5})$
2. $(-\frac{1}{4}) + (+\frac{1}{8})$
3. $(+\frac{3}{5}) + (-\frac{7}{10})$
4. $(-\frac{8}{9}) + (+\frac{5}{6})$

C. Subtract the following: Note to teacher: (Students will probably need much practice with this material,)

1. $\frac{2}{3} - \frac{1}{7}$
2. $12\frac{3}{5} - 8\frac{1}{2}$
3. $4 - \frac{3}{15}$
4. $8\frac{3}{8} - 2\frac{1}{2}$

SUMMARY:

Fabric Warehouse has a 26-yd bolt of purple silk. Mary Ann purchased $3\frac{1}{4}$ yds, Bonita bought $2\frac{3}{8}$ yds, and Kim bought $4\frac{1}{2}$ yds. How much material remained?

$(+26) - (+3\frac{1}{4} + 2\frac{3}{8} + 4\frac{1}{2}) = ?$

UNIT II: INTRODUCTION TO ALGEBRA

SUGGESTED TEST ITEMS

Note: This unit is extremely long. The teacher should give several sub-tests during the unit. This post-test mixes many objectives in each question.

1. Use the correct symbol $<$, $=$, or $>$ to make each statement true.

a. $+9$ _____ $+3$

b. -7 _____ $-1\frac{2}{3}$

c. $\frac{3}{4}$ _____ $\frac{9}{12}$

d. $\frac{5}{9}$ _____ $\frac{2}{3}$

e. $-\frac{3}{5}$ _____ $-\frac{9}{7}$

f. -11 _____ $+5$

g. $(+12) + (-9)$ _____ $(-9) + (+12)$

h. 5×3 _____ $25 - 11$

2. Which number in each group will appear furthest to the right on a number line?

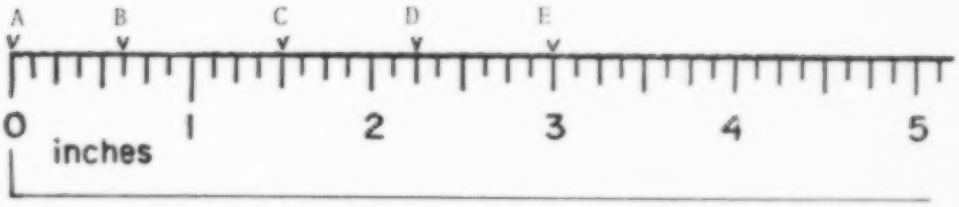
a) $-2, +7, 0$

b) $-9, -3, -1$

c) $-\frac{2}{3}, -\frac{9}{10}, +\frac{1}{3}$

d) $\frac{2}{7}, \frac{2}{9}, \frac{2}{3}$

*3.



Write the number in column II that corresponds to the item given in column I.

COLUMN I	COLUMN II
A	$\frac{1}{4}$
B	$\frac{1}{2}$
D	
E	$\frac{5}{8}$
Length of $\overline{AC}$	0
	$2\frac{1}{4}$
	$2\frac{1}{8}$
	3
	$1\frac{1}{4}$
	$1\frac{1}{2}$

4. Write the signed number or signed numbers described by each of the following statements.

   a. 5 units to the right of +3

   b. 9 units to the left of +2

   c. A bank withdrawal of $100

   d. The change in temperature from -2° to $+5^{\circ}$

   e. The opposite of -12.

   f. The numbers whose absolute value is 7.

   g. The additive inverse of +15

*5. Compute the following

a) (+5) + (+15)

b) (-7) + (-9)

c) (-15) + (+21)

d) (+43) + (-17)

e) Add +19

-42

*6. Which number has the same value as $\frac{19}{6}$?

a) 13 b) $2\frac{1}{6}$ c) $3\frac{1}{6}$ d) $6\frac{1}{3}$

*7. Which number has a value between $\frac{1}{2}$ and $\frac{3}{4}$?

a) 1 b) $\frac{2}{3}$ c) $\frac{11}{12}$ d) $\frac{4}{5}$

*8. What is the least common denominator of $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{5}$?

a) 6 b) 10 c) 15 d) 30

*9. Wendy had 5 yards of material. If she used $3\frac{3}{4}$ yards, how many yards does she have left?

a) $1\frac{1}{4}$ b) $1\frac{3}{4}$ c) $2\frac{1}{4}$ d) $2\frac{3}{4}$

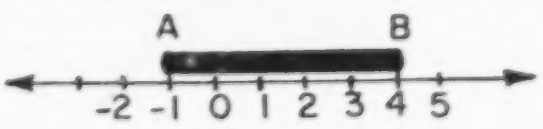
*10 What is the sum of $\frac{5}{7}$ and $\frac{2}{5}$?

*11. Subtract $\frac{3}{4}$ from $1\frac{1}{2}$

*12. Subtract $5\frac{2}{7}$

$- 2\frac{3}{7}$

*13. What is the length of this line segment joining points A and B ?



Unit III. Metric Measurement

LESSON 1

AIM: To introduce the units of linear measurement in the metric system

PERFORMANCE OBJECTIVES: The student will be able to:

- identify the meter as the basic unit of linear measure in the metric system
- express the numerical relationships between the meter and the millimeter, the centimeter, and the kilometer.
- recognize (visually) the similarity of the meter and the yard without making conversions.
- recognize the relative sizes of and indicate appropriate uses for the millimeter, centimeter and kilometer.

VOCABULARY:

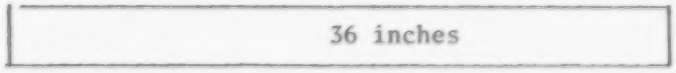
New Terms: metric system, meter, kilometer, centimeter, millimeter, prefix

CHALLENGE PROBLEM:

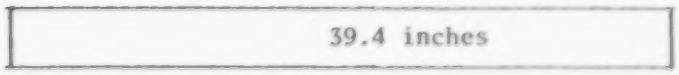
In the Olympic Games, one race is a 100-meter dash. Is this distance more or less than the length of a football field (100 yards)?

DEVELOPMENT:

1. Introduce the term "meter" as the basic unit of linear measure in the metric system. To provide proper perspective for the class, the teacher should sketch the following diagram on the board:



yard stick



meter stick

Other visual aids, such as a meter stick and a yard stick, may be used for comparison purposes only. (Note to teacher: This unit will not be concerned with conversions from metric to English units or vice versa, but rather with conversions only within the metric system.)

2. Pupils should now be able to answer the challenge problem. Since the length of a football field (from goal post to goal post) is 100 yards, and one meter is longer than one yard, 100 meters is a greater distance than 100 yards.
3. Discuss the basic prefixes of linear measure in the class, indicating their usual abbreviations, and the relationship between each unit and the meter. The following table will be helpful:

unit	abbreviation	length
kilometer	km	1000 meters
hectometer	hm	100 "
dekameter	dkm	10 "
meter	m	1 meter
decimeter	dm	$\frac{1}{10}$ "
centimeter	cm	$\frac{1}{100}$ "
millimeter	mm	$\frac{1}{1000}$ "

Note that the abbreviations are <u>not</u> followed by periods. Indicate to the pupils that the most commonly used units are the kilometer, meter, centimeter, and millimeter. These are the only ones that will be considered in this unit.

APPLICATIONS:

A. Help pupils to understand the relative sizes of the kilometer, meter, centimeter and millimeter, by characterizing the kilometer as a large unit for measuring long distances, the meter as a unit of moderate size, and the centimeter and millimeter as small units.

B. Place a check in the box which indicates the most suitable unit for measuring each of the following situations:

Situation	mm	cm	m	km
1. Distance between two cities				
2. Length of a classroom				
3. Height of a pupil				
4. Length of a pencil				
5. The width of a film negative				

C. Choose the most logical measure of each item from the choices given.

1. The width of this sheet of paper.
   20 mm 20 cm 20 m 20 km
2. The length of an automobile.
   5 mm 5 cm 5 m 5 km
3. The height of the World Trade Center Towers.
   0.4 mm 0.4 cm 0.4 m 0.4 km
4. The thickness of a dime.
   1 mm 1 cm 1 m 1 km
5. The length of a newborn baby.
   40 mm 40 cm 40 m 40 km

SUMMARY:

Review vocabulary and relative magnitudes of the metric units discussed.

LESSON 2

AIM: To read and interpret large numbers and decimals

PERFORMANCE OBJECTIVES: The student will be able to:

- write a numeral given in verbal form
- name the place value of any digit in a whole number up to the millions place
- name the place value of any digit in a decimal number up to the ten thousandths place
- read any number expressed in symbol form from .0001 to 9,999,999 in words and vice versa
- define negative exponents for powers of 10
- express a fraction whose denominator is 10,000, or 1000 as a decimal or vice versa

VOCABULARY:

New Terms: numeral, digit, common fraction, decimal fraction, negative exponent, place value

Review: denominator, power, lowest terms.

CHALLENGE PROBLEM:

Write as a numeral two million, eleven thousand, five hundred three.

DEVELOPMENT:

1. Discuss challenge problem. Why can't we write 21,153? Lead into discussion of <u>place value</u>, making sure to express places as powers of 10 as follows:

1,000,000	...	100	10	1
10^6		10^2	10^1	10^0
Millions		Hundreds	Tens	Units

2. Do applications A and B.
3. What is the meaning of 1.6? Use pattern approach to extend place values of 10 to include <u>negative exponents</u>.

1,000,000	...	100	10	1	$\frac{1}{10}$	$\frac{1}{100}$
10^6	...	10^2	10^1	10^0	10^{-1}	10^{-2}
				Units	Tenths	Hundredths
					.1	.01

Discuss the place values to the right of the decimal point. Emphasize that each place has a name which ends in "th." Each decimal has a first and last name.

The first name is given by reading the decimal portion as if there were no decimal point. The last name is given by reading the place value of the right-most digit in the numeral.

Examples: 0.47 is read $\underbrace{\text{Forty-seven}}_{\text{First Name}}$, $\underbrace{\text{Hundredths}}_{\text{Last Name}}$

0.051 is read $\underbrace{\text{Fifty-one}}_{\text{First Name}}$, $\underbrace{\text{Thousandths}}_{\text{Last Name}}$

When digits appear to the right and left of a decimal point, the decimal point is read "and".

Example: 3.09 is read Three and $\underbrace{\text{Nine}}_{\text{First Name}}$, $\underbrace{\text{Hundredths}}_{\text{Last Name}}$

4. Do applications C and D.

5. Express decimal fractions as common fractions whose denominators are powers of 10 and vice versa.

Examples: 1. Express $\frac{42}{10000}$ as a decimal fraction.

i. Read this as 42 ten thousandths.

ii. Write this as 0.0042 (Note: Have the students write 42 and then position the decimal point so that 2 is in the ten thousandths place.

2. Express 0.203 as a common fraction.

i Read 0.203 as two-hundred-three thousandths.

ii Write this as $\frac{203}{1000}$. (All answers should be expressed in lowest terms.)

6. Do applications E and F.

APPLICATIONS:

A. Write in symbols and in words the numeral that contains:

1. 9 in the tens place
   0 in the hundreds place
   3 in the ones place
   2 in the thousands place
2. 0 in the hundreds place
   0 in the thousands place
   3 in the ten thousands place
   5 in the ones place
   2 in the tens place

B. What is the value of the 7 in each of the following numerals?

1. 507
2. 2,713
3. 72,300
4. 576

C. Write each of the following in words:

1. 0.8
2. 0.23
3. 0.04
4. 2.5
5. 17.001

D. Write each of the following in decimal notation:

1. fifteen hundredths
2. two hundred seventy-five thousandths
3. eighty-seven thousandths
4. nine and six tenths
5. three and two hundredths

E. Express each common fraction as a decimal fraction:

1. $\frac{3}{10}$
2. $\frac{3}{100}$
3. $\frac{74}{100}$
4. $\frac{74}{1000}$
5. $\frac{95}{10,000}$

F. Express each decimal fraction as a common fraction.

1. 0.7
2. 0.07
3. 0.99
4. 0.013
5. 0.0035
6. 1.35

SUMMARY:

For the number 327.461,

1. indicate the place value of each digit.
2. write the place value of each digit as a power of 10.

LESSON 3

AIM: To develop understanding of the metric ruler and the relationship between the meter and the centimeter

PERFORMANCE OBJECTIVES: The student will be able to:

- identify and read the centimeter markings on a metric ruler
- express the subdivisions of a centimeter as tenths of a centimeter
- measure lengths in meters and centimeters with a ruler
- apply the rule for converting meters to centimeters by multiplying by 100

VOCABULARY:

New Terms: meter stick

Review: centimeter, meter, line segment and its notation ($\overline{AB}$)

CHALLENGE PROBLEM:

John is 1.6 m tall and Harold is 165 cm tall. Who is taller?

DEVELOPMENT:

1. Discuss challenge problem. Elicit a need to convert from meters (m) to centimeters (cm).
2. Have the class recall the magnitude of the meter. Have the pupils use the meter stick to measure the lengths of common objects in the classroom, such as the classroom itself, the teacher's desk, the chalkboard, windows, etc., whose lengths would commonly be expressed in meters. Such measurements may be made to the nearest meter, and the students should record them in their notes.
3. Show the pupils that the meter stick is divided into 100 smaller units called "centimeters" which can be used to measure objects that are smaller in size than a meter. Review the meaning of the prefix "centi" comparing it to the word "cent," meaning hundredth of a dollar. Introduce the basic relationship, 1 meter (m) = 100 centimeters (cm). Elicit that in order to change m to cm, multiply the number of meters by 100.

Note: A review of the procedure for multiplying any number by 100 is usually necessary. Stress the proper placement of the decimal point in whole numbers and in decimal fractions. Multiplying by 100 has the effect of moving the decimal two places to the right.

4. Develop a table with the pupils, such as the following:

   0.1 meter = 10 centimeters

   0.2 meter = 20 centimeters

   etc.

   0.9 meter = 90 centimeters

   1.0 meter = 100 centimeters

5. Do application A.

6. Return to challenge problem. The pupil should now be able to convert 1.6 m to 160 cm, which is less than 165 cm.

7. Introduce measuring smaller items using a centimeter ruler; i.e., divide each centimeter (cm) into 10 equal parts and express the subdivisions as tenths (.1) of a centimeter.

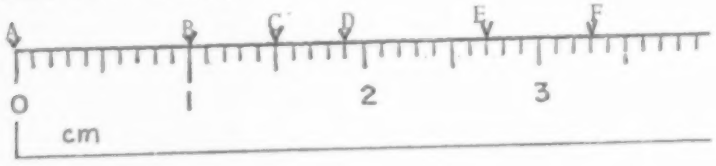
8. Do application B.

APPLICATIONS:

A. Convert each measurement to centimeters:

2 m	0.32 m
20 m	1.4 m
35 m	3.02 m
0.7 m	0.001 m

B. Express, in meters, the length of each line segment. (Use decimal fractions.)



$\overline{AC}$ = $\overline{BD}$ = $\overline{CE}$ =

$\overline{AB}$ = $\overline{AD}$ = $\overline{BF}$ =

C. Measure the following line segments to the nearest tenth of a centimeter:

A_____B

C_____ _____D

E_____F

D. Using a centimeter ruler, measure various items in the room such as:

1. length of a 12-inch ruler
2. length and width of an I.D. card or bus pass
3. width of a desk

SUMMARY:

A yardstick measures approximately 0.9 meters. Convert this measure to centimeters.

CLIFF-HANGER:

Measure the diameter of a nickel in centimeters and record the result in your notebook. Measure the thickness of the nickel. What problem, if any, did you have when measuring its thickness?

(Note to teacher: Explain meaning of "diameter," if necessary.)

LESSON 4

AIM: To develop understanding of the relationship between the centimeter and the millimeter, and between the meter and the millimeter.

PERFORMANCE OBJECTIVES: The student will be able to:

- identify and read the millimeter markings on a ruler
- convert measurements from cm to mm and m to mm (multiply by 10 and 1000).
- measure lengths in millimeters

VOCABULARY:

Review: meters, centimeters, millimeters, power

CHALLENGE PROBLEM:

What is the measure of the thickness of a nickel?

DEVELOPMENT:

1. Discuss the challenge problem and the cliff-hanger. Students should recognize that the centimeter was not a small enough unit to measure the nickel, and that they had to use tenths of a centimeter to measure accurately. Students should be told that a tenth of a centimeter is called a "millimeter."

   Thus: 0.1 cm = 1 mm
   and
   0.2 cm - 2 mm

   Using a development similar to the one used for reviewing multiplication by 100, review multiplication by 10.

2. Do application A.

3. From the relationships 1 cm = 10 mm and 1 m = 100 cm, elicit the relationship between m and mm (1 m = 1000 mm). Review prefix "milli" from a previous lesson and discuss the procedure for multiplying by 1000.

4. Do application B.

5. Summarize: To change from a larger to smaller unit, it is necessary to multiply by the appropriate power of 10. Review multiplication by 10, 100, 1000.

6. Do application C.

APPLICATIONS:

A. Convert the following centimeter measurements to millimeters:

2 cm	0.32 cm
20 cm	1.6 cm
35 cm	3.04 cm
0.8 cm	3.2 cm

B. Convert the following meter measurements to millimeters:

9 m	0.27 m
8.6 m	0.05 m
0.3 m	

C. Change each measurement to the indicated unit:

9 m = _____ cm	0.498 cm = _____ mm
8.8 m = _____ mm	0.498 m = _____ cm
0.9 cm = _____ mm	0.498 m = _____ mm
0.38 m = _____ cm	

SUMMARY:

Elicit the following rules:

1. To change from a larger unit of measure to a smaller one in the metric system, multiply by the appropriate power of ten (10, 100, 1000, etc.).

2. To multiply by a power of ten, move the decimal point to the right the appropriate number of places.

LESSON 5

AIM: To convert from smaller to larger metric units

PERFORMANCE OBJECTIVES: The student will be able to:

- convert from millimeters to centimeters, from centimeters to meters and from millimeters to meters.

VOCABULARY:

Review: centimeter, millimeter, meter

CHALLENGE PROBLEM:

Find the number of centimeters in each of the following:

(a) 2.5 m (b) 25 mm

DEVELOPMENT:

1. Discuss challenge problem. Students should have no difficulty with part (a) but should discover they haven't learned to change a measurement from a small unit to a larger unit.

2. One method of finding a solution to the problem in part (b) is to use a cm ruler. Students should see that 25 mm markings correspond to 2.5 cm.
   Elicit the arithmetic process used to make this conversion (division by 10). Teach division by 10 using an approach similar to the one that was used for teaching multiplication by 10.

3. Do application A.

4. In a similar manner develop the relationships
   $1 \text{ cm} = \frac{1}{100} \text{ m} = .01 \text{ m}$ and $1 \text{ mm} = \frac{1}{1000} \text{ m} = .001 \text{ m}$
   and apply to conversions such as:
   $25 \text{ cm} = \frac{25}{100} \text{ m} = 0.25 \text{ m}$ $35 \text{ mm} = \frac{35}{1000} \text{ m} = 0.035 \text{ m}$
   Review division of a whole number or a decimal by 100 and 1000.

5. Do application B and C.

APPLICATIONS:

A. Find the number of centimeters in each measure.

38 mm	250 mm	6.25 mm
2 mm	4155 mm	1.05 mm
2.5 mm		

B. Change each measure to the indicated unit.

87 cm = _____ m	257 mm = _____ m
786 cm = _____ m	8.7 mm = _____ m
3 mm = _____ cm	8.7 cm = _____ m
97 mm = _____ cm	

SUMMARY:

Construct a conversion chart showing the relationship between the metric units, starting with 7 meters.

7			
METER	DECIMETER	CENTIMETER	MILLIMETER

Note: Decimeter is placed in the chart to maintain the place value but is not an important measure.

Review that in order to convert from one unit to another, moving to the left, divide by 10, 100, or 1000 depending upon the number of spaces moved.

The teacher should also review changes to the right as a multiplication by 10, 100, or 1000 depending upon the number of spaces moved.

CLIFF-HANGER:

New car speedometers have two sets of numbers on the face of the dial. Why do the numbers 80 and 50 appear at the same point on the dial?

LESSON 6

AIM: To develop understanding of the relationship between the meter and the kilometer

PERFORMANCE OBJECTIVES: The student will be able to:

- compare the relative sizes of kilometers and miles, e.g., 50 miles is about 80 kilometers.
- convert measurements from meters to kilometers and vice versa

VOCABULARY:

Review: kilometer, meter

CHALLENGE PROBLEM:

A road sign on a main U.S. highway now reads: "Speed Limit: 88 kilometers per hour" (88 km/hr). The law limits the speed of a car anywhere in the U.S. to a maximum of 55 miles per hour. How does 88 km/hr compare to the U.S. speed limit?

DEVELOPMENT:

1. Begin the discussion with the challenge problem and the cliff-hanger. Show the class a picture of a typical metric road sign:

Mention to the pupils the fact that most American road signs also include speed limits in miles per hour, and that 88 km is approximately equivalent to 55 miles. (This conversion information is provided for perspective only, not for instruction.) Point out to the class the fact that the meter is too small a unit to use for the measurement of the distance between two cities. A common unit for measuring large distances is the "kilometer" (km) which is equal to 1000 meters.

2. Discuss the meaning of the prefix "kilo" (one thousand), and mention other units using this prefix with which the pupils may be familiar: kilowatt, kilowatt hour (measures of electrical power).
   Elicit that to convert from the larger unit, kilometer, to the smaller unit, meter, we multiply by 1000.

3. Develop a table with the pupils, such as the following:
   0.1 km = 100 meters
   0.2 km = 200 meters
   etc.
   0.9 km = 900 meters
   1.0 km = 1000 meters

4. Do application A.

5. If John and Mary live 500 meters apart, have students express this distance in kilometers. What operation must be performed to change meters to kilometers? To change from a smaller unit, (meter) to a larger unit (kilometer), divide by 1000.

6. Do application B.

APPLICATIONS:

A. Convert each distance given to meters:

1. 3 km	4. 3.25 km
2. 25 km	5. 0.675 km
3. 7.5 km	6. 236 km

B. Convert each distance given to kilometers:

1. 6000 m	4. 7 m
2. 575 m	5. 0.5 m
3. 32.5 m	

SUMMARY:

1. If a pupil walks 1.5 kilometers to school, and the same distance home each day, how many meters does he walk?

2. One of the longer races in the Olympic Games is 5000 meters. How many kilometers is this distance?

LESSON 7

AIM: To provide practice in converting from one metric unit of linear measure to another, with a variety of application exercises

INSTRUCTIONAL OBJECTIVES: The student will be able to:

- apply the metric lengths to various problem situations.
- express the quotient of two whole numbers as a decimal.

VOCABULARY:

Review: kilometer, meter, centimeter, millimeter

CHALLENGE PROBLEM:

A carpenter wishes to cut a meter board into eight equal parts. Express the length of each part in meters, centimeters, and millimeters.

DEVELOPMENT:

1. Discuss challenge problem. Divide 1.000 m by 8 so that students are introduced to the concept of dividing a smaller by a larger number. Note that computations will be simplified if they divide 1000 mm by 8 and convert to centimeters and meters.
2. Review the procedure for dividing two whole numbers, expressing the quotient as a decimal. For example:

$$\frac{3}{4} = 4\overline{)3.00}\quad .75$$

$$\begin{array}{r} 28 \\ \hline 20 \\ 20 \\ \hline \end{array}$$

The teacher should provide additional practice.

3. Do applications.

APPLICATIONS:

A. Express each quotient as a decimal.

1. $4 \div 5 =$
2. $1 \div 2 =$
3. $\frac{7}{8}$
4. $\frac{15}{4}$
5. $1 \div 25$

B. 1. Jose is making a bookshelf. How many meters of lumber should he purchase if he wants to put up 6 shelves each 90 cm long?

2. Teddy is moving his water heater from the front to the rear of his basement. He has to extend the water pipe across the ceiling of his basement. How many 75 cm sections of pipe will he need to cross a ceiling 13.5 meters long?

3. Make the following conversions:

(a)	7.21 km = _____ m	(b)	35 mm = _____ cm
(c)	842 cm = _____ m	(d)	71 m = _____ mm
(e)	61.3 cm = _____ mm	(f)	753 m = _____ km
(g)	5.32 cm = _____ m	(h)	47.7 cm = _____ mm
(i)	28.4 km = _____ m		

SUMMARY:

Review the method of converting different metric units of linear measure into each other. Extend chart constructed in lesson 5 to include kilometers.

LESSON 8

AIM: To introduce the units of measure of weight in the metric system

PERFORMANCE OBJECTIVES: The student will be able to:

- state that the gram is the basic unit of weight in the metric system
- state that there are "about 30 grams to one ounce" (no conversions from one system to another are to be made)
- express the relationships among grams, milligrams and kilograms
- compare the relative sizes of, and indicate appropriate uses for, the milligram, gram, and kilogram

VOCABULARY:

New Terms: net weight, gram, kilogram, milligram

Review: prefixes, kilo, milli

CHALLENGE PROBLEM:

A box of rice is marked "Net Weight: 340 grams". Is this amount more or less than one pound?

DEVELOPMENT:

1. Introduce the term "gram" as the basic unit of measure of weight in the metric system. The class should be told that a gram is a very small unit of weight. A paper clip weighs about one gram. To provide additional perspective, the pupils may be told that one ounce is equivalent to approximately 28.35 grams. A metric scale may be used to weigh common objects found in the classroom, such as a pencil, a pen, a piece of chalk, coins, etc., whose weights would be small in terms of grams. (Note to teacher: This unit will not be concerned with conversions from metric to English units or vice versa.)

2. Help the pupils to answer the challenge problem now, as follows: If 28.35 grams = 1 ounce and 16 ounces = 1 pound, then a metric equivalent for 1 pound can be found by multiplying 28.35 x 16 = 454 grams. Therefore, 340 grams is less than one pound. (It is

about 12 ounces or 3/4 of one pound).

3. Discuss the important prefixes of weight measure in the metric system, indicating their usual abbreviations, and the relationship between each unit and the gram. The following table will be helpful:

unit	abbreviation	weight
kilogram	kg	1000 grams
GRAM	g	1 gram
milligram	mg	0.001 gram

Note that the abbreviations are not followed by periods.

APPLICATIONS:

1. Help pupils to understand the relative sizes of the kilogram, gram and milligram, by characterizing the kilogram as a unit of moderate size (in which the weight of a pupil would be expressed, for example), the gram as a fairly small unit, and the milligram as a very small unit (commonly used to measure ingredients of medicines or vitamins).

2. Place a check in the box which indicates the most suitable unit for measuring the weight of the following:

Object	kg	g	mg
1. A heavy bag of potatoes			
2. The weight of a penny			
3. The amount of vitamin C in a teaspoonful of vitamin drops			
4. The weight of a stick of chalk			
5. The weight of a pupil			

SUMMARY:

Review the new vocabulary introduced in this lesson and the relative magnitudes of the metric units discussed.

CLIFF-HANGER:

One bag of potatoes weighs 1.6 kilograms and another bag weighs 1650 grams. Which bag is heavier?

LESSON 9

AIM: To further develop understanding of the relationship between the gram and the kilogram

PERFORMANCE OBJECTIVES: The student will be able to:

- convert from grams to kilograms and vice versa.

VOCABULARY:

Review: kilogram, gram, milligram

CHALLENGE PROBLEM:

One bag of potatoes weighs 1.6 kilograms, and another bag weighs 1650 grams. Which bag is heavier?

DEVELOPMENT:

1. Explain to the class that, since the gram is such a small unit of weight, the unit used most commonly in the market is the "kilogram." The basic relationship is 1 kilogram (kg) = 1000 grams (g).

   Review the meaning of the prefix "kilo." To provide perspective, pupils may be reminded that in lesson 8 it was observed that 454 grams was approximately equivalent to 1 pound; so 1 kg, or 1000 grams, would be a little more than 2 pounds (actually about 2.2 pounds).

2. Develop this table:

   0.1 kg = 100 g
   0.2 kg = 200 g
   etc.
   0.9 kg = 900 g
   1 kg = 1000 g

3. The students should now be able to answer the question in the challenge problem: Since 1.6 kg = 1000 g + 600 g = 1600 g, then 1.6 kg is less than 1650 g.

APPLICATIONS:

1. Some common objects in the classroom (such as a textbook, a pile of notebooks, mathematical models or kits in the room, etc.) may be weighed in kilograms on a metric scale. All results should be recorded in kilograms on a chart as shown. The teacher should add other items to the chart.

Object	Weight in kilograms
Mathematics textbook	
Loose leaf notebook	
Any skills development kit in the classroom	
A pair of shoes	
Different pupils' weights	

2. Some of the weights measured in exercise 1, above, may be converted to grams by multiplying by 1000, and weights measured earlier in grams (lesson 8) may be converted to kilograms by dividing by 1000.

SUMMARY:

Review the basic relationship between the gram and the kilogram, and the types of objects that might be weighed using each unit.

CLIFF-HANGER:

A bottle of vitamin tablets indicates that each tablet contains 250 mg of Vitamin C. How many tablets would a person have to take to obtain 1 gram of Vitamin C?

LESSON 10

AIM: To develop further understanding of the relationship between the gram and the milligram

PERFORMANCE OBJECTIVES

Students will be able to convert from grams to milligrams and vice versa.

VOCABULARY:

Review: All metric prefixes.

CHALLENGE PROBLEM:

Begin the discussion with the cliff-hanger problem from the previous lesson.

DEVELOPMENT:

1. Explain to the pupils that, although the gram is itself a small weight, much smaller weights are needed in such fields as medicine, chemistry, and other sciences to measure very small quantities of chemicals, etc. Such small weights are expressed in "milligrams." The basic relationship is 1 gram = 1000 milligrams. 1 g = 1000 mg or 1 mg = .001 g. Review the meaning of the prefix "milli."
2. Develop this table:

   0.1 g = 100 mg

   0.2 g = 200 mg

   etc.

   0.9 g = 900 mg

   1.0 g = 1000 mg
3. Return to the challenge problem. The pupils should now be able to answer this question: 250 mg = 0.25 g (or 1/4 gram), so 1 gram = 4 x 0.25 gram. Therefore, 4 tablets need to be consumed to obtain 1000 mg or 1 g of Vitamin C.

APPLICATIONS:

1. If one teaspoonful of vitamin drops contains 50 mg of Vitamin C, how many milligrams would be contained in 3 teaspoonsful of the drops? In 5 teaspoonsful? How many teaspoonsful would contain 1 gram of Vitamin C? 2 grams?
2. A chemist weighed 0.7 gram of a certain chemical for an experiment. How many milligrams is this?
3. A certain chemical compound contained 450 milligrams of a particular chemical. What part of a gram is this?
4. Practice may be provided in converting milligrams to grams by dividing by 1000, or in converting grams to milligrams by multiplying by 100.

SUMMARY:

Review the basic relationship between the gram and the milligram and indicate for different objects the most appropriate unit for weighing.

UNIT III: METRIC MEASUREMENTS

SUGGESTED TEST ITEMS:

1. Choose the most reasonable measure for each object.

   a) Height of the World Trade Towers

      405 mm 405 cm 405 m 405 km

   b) Distance from Detroit to Cincinnati

      400 mm 400 cm 400 m 400 km

   c) Thickness of a nickel

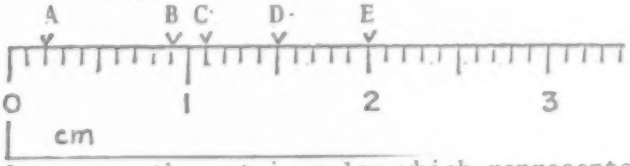
      2 mm 2 cm 2 m 2 km

   d) The weight of a vitamin pill

      250 mg 250 g 250 kg

   e) The weight of a 19 inch portable television set

      28 mg 28 g 28 kg

2. 

   Choose the letter on the metric ruler which represents each measure.

   a) 0.9 cm c) 2 cm

   b) 15 mm d) 2 mm

3. Change each metric measure to its equivalent in the indicated unit.

   a) 280 cm = _____ m g) 17.2 m = _____ cm

   b) 763 m = _____ km h) 3700 mg= _____ g

   c) 5.1 km = _____ m i) 0.4 g = _____ mg

   d) 0.9 cm = _____ mm j) 250 g = _____ kg

   e) 5 m = _____ mm k) 2 kg = _____ g

   f) 73,000 mm= _____ m

*4. Write the numeral for thirty-six thousand, four hundred eighty.

5. In the number 3762 the value of the 7 is

   a) 7×10^0 b) 7×10^1 c) 7×10^2 d) 7×10^3

6. In the number 102.34 which digit is in the hundredths place?

   a) 1 b) 2 c) 3 d) 4

*7. What number represents twenty-six and seven tenths?

a) 0.267 b) 26.07 c. 26.7 d) 267

8. Which decimal number is equivalent to the common fraction $\frac{37}{1000}$?

a) 37000 b) 37 c) 0.37 d) 0.037

*9. How many centimeters are equal to 2 meters?

a) 20 b) 2 c) 200 d) 2000

Unit IV. Plane Geometry

LESSON 1

AIM: To identify triangles and to classify them according to the lengths of their sides

PERFORMANCE OBJECTIVES: Students will be able to

- illustrate the importance of geometric shapes by referring to their uses in architecture and elsewhere.
- state the definition of triangle.
- divide the set of triangles into three groups: scalene, isosceles, and equilateral, relating the classification to the number of equal sides.

VOCABULARY:

New Terms: rigid, triangle, scalene, isosceles, equilateral

CHALLENGE PROBLEM:

Which of these geometric figures would be considered a "rigid figure"? (A rigid figure is one that will not change its shape under reasonable pressure from any direction.

RECTANGLE

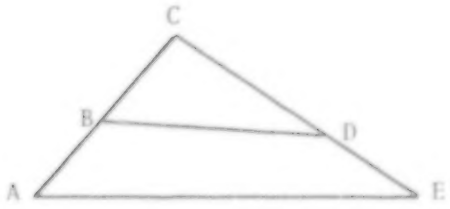
TRIANGLE

DEVELOPMENT:

1. Explore the values of a rigid figure in construction, such as in bridge building, in the manufacture of furniture, etc. Demonstrate to the class with an extensible quadrilateral how a rectangle can be deformed into a parallelogram by pressure. Show how this can be prevented by the introduction of a diagonal, creating triangles.
2. Define the word "triangle" as a closed figure having three sides.
3. Draw and classify triangles according to the lengths of their sides. A "scalene" triangle has three sides of unequal length. An "isosceles" triangle has two equal sides (legs) and a third side (base). An "equilateral" triangle has all three sides of equal length.

4. Review the method of naming line segments using upper case letters to name the end points and giving students practice in naming triangles.

Example:



This diagram contains two triangles.

i. Triangle BCD (ΔBCD) formed by line segments BC, CD, and BD.

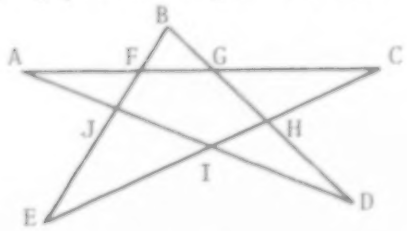
ii. Triangle AEC (ΔAEC) formed by line segments AE, EC, and CA.

APPLICATIONS:

A. Draw scalene and isosceles triangles with different shapes and different orientations so students may observe the different appearances such triangles may assume.

B. Draw equilateral triangles of different sizes so that students may observe that they all have the same shape and that the variations in shape possible with scalene and isosceles triangles are not possible with equilateral triangles.

C. Prepare a rexographed sheet and an overhead transparency of a variety of triangles and have the students classify each triangle as scalene, isosceles, or equilateral. In some cases, the measures may be listed on the diagrams and, in other cases, the students should use rulers to measure the sides.
A very challenging problem might be added as follows:



Name all the triangles in this figure and classify each triangle according to the lengths of its sides.

SUMMARY:

1. Elicit a review of the new vocabulary introduced in this lesson.
2. Test students' understanding with such questions as the following:
   (a) Is an equilateral triangle also isosceles?
   (b) Is an isosceles triangle also equilateral?

LESSON 2

AIM: To define "angle" and types of angles and to measure and label angles

PERFORMANCE OBJECTIVES: Students will be able to

- describe an angle as a rotation (as the angle between the hands of a clock).
- state that one complete rotation is divided into 360 units called degrees.
- state that one-quarter of a rotation is a right angle (90°).
- state that one-half of a rotation is a straight angle (180°).
- state a definition for an acute angle, a right angle, an obtuse angle, and a straight angle.
- select by sight from given diagrams an acute, right, obtuse, and straight angle.
- to find the measure of an angle using a protractor.

VOCABULARY:

New Terms: degree, acute, obtuse, right, straight, protractor, vertex

CHALLENGE PROBLEM:

In order to keep a ladder from slipping, a man placed it against the side of a building so that it formed an angle of 70° with the ground. Sketch a diagram illustrating this situation.

DEVELOPMENT:

1. Discuss the challenge problem. Students will generally have a vague idea of the approximate size of a 70° angle. Use this vagueness as a springboard to begin the following development. The size or measure of an angle depends on the amount of rotation and not on the lengths of the sides. For example, in Figure A the side $\overrightarrow{OB}$ (a ray) has been rotated from its initial position, $\overrightarrow{OA}$, to the terminal position, $\overrightarrow{OB}$. The ray has moved $\frac{1}{4}$ of a revolution. One complete revolution is 360° (read "360 degrees"). The angle is $\frac{1}{4}$ of 360° or 90°. This angle is called a "right angle." In Figure B, the line segment has moved $\frac{1}{2}$ of a revolution, $\frac{1}{2}$ of 360° or 180°. This angle is called a "straight angle." (Illustrate the formation of angles with objects of

different lengths for their terminal and initial sides. For example, the angle formed by the hands of a wristwatch, a school clock, and Big Ben, when it is 3:00, is a right angle.)

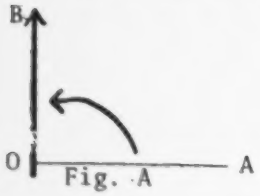


Fig. A

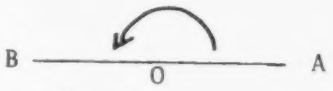


Fig. B

2. Demonstrate different types of angles by rotating the ray $\overrightarrow{OA}$ from figure A. (A board compass is excellent for this demonstration.)
3. Define "angle," "acute angle," "obtuse angle," "right angle," and "straight angle" in terms of both rotation and degree measure.
4. Demonstrate the different ways of naming an angle using letters:

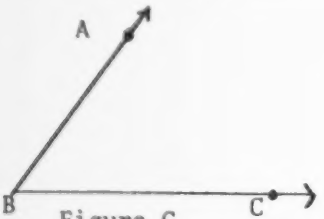


Figure C

In Figure C the angle may be named $\angle ABC$ or $\angle CBA$. Stress that the vertex is listed in the middle of the 3 letters. $\overrightarrow{BA}$ and $\overrightarrow{BC}$ are called the sides of the angle.

Where only one angle is formed at a particular vertex, the angle may be named with a single letter, $\angle B$.

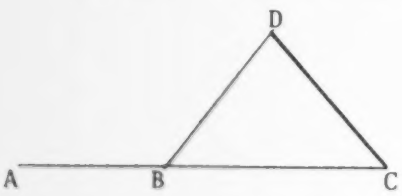


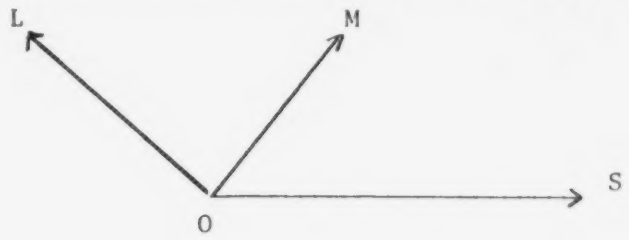
Figure D

In Figure D, $\angle BDC$ may be named $\angle D$ and $\angle BCD$ may be named $\angle C$ but $\angle DBC$ may not be called $\angle B$ because there are three angles with a vertex at B, namely, $\angle ABC$, $\angle DBC$, and $\angle ABD$.

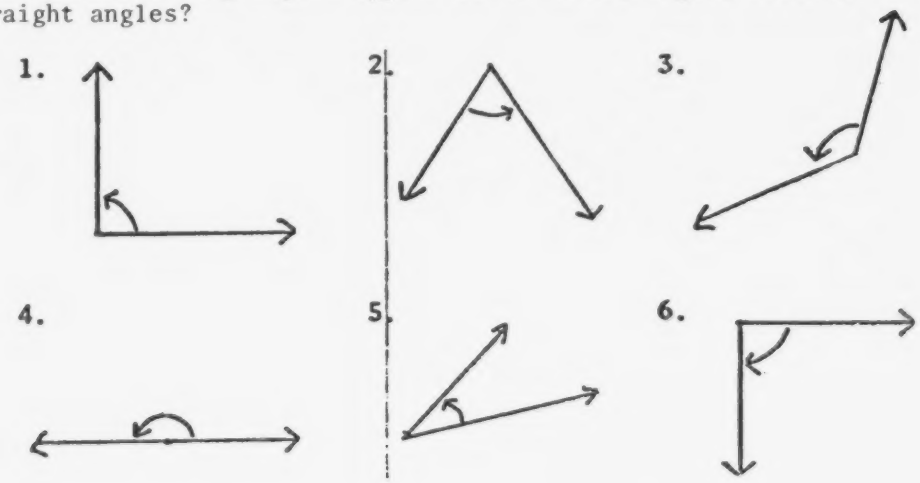
5. Do applications A and B.
6. Demonstrate to pupils how a protractor is used to find the measure of an angle. Emphasize that the vertex of the protractor must be placed on the vertex of the angle and that the side of the protractor (the zero degree line) must lie on one side of the angle.
7. Do application C.

APPLICATIONS:

A. Name all the angles on the diagram, by letter.



B Which of the following angles appear to be acute, right, obtuse, or straight angles?



C. Use a protractor to measure several different angles drawn on rexographed sheets.

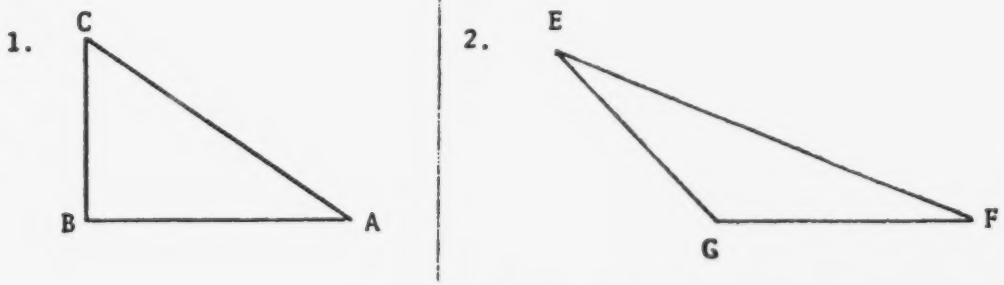
SUMMARY:

In the following triangles:

a) describe the angles as acute, obtuse, or right

b) estimate the measures of the angles

c) use a protractor to find their actual measurement



LESSON 3

AIM: To determine the sum of the measures of the angles of a triangle

PERFORMANCE OBJECTIVES: The student will be able to:

- state the relationship that the sum of the measures of the angles of a triangle is 180^{o}.
- apply this relationship to find the measure of the third angle of a triangle when the measures of two angles are given,
- state that a triangle cannot include two right angles, two obtuse angles, or a right angle and an obtuse angle.

VOCABULARY:

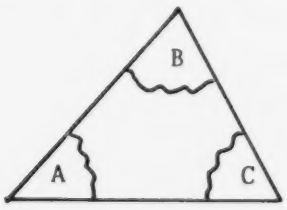
Review: triangle, degrees, obtuse, acute

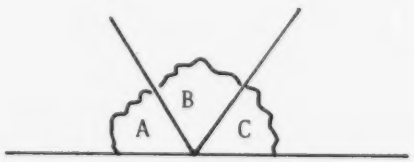
CHALLENGE PROBLEM:

Draw a triangle and measure its angles. What is the sum of the measures of these three angles?

DEVELOPMENT:

1. Discuss challenge problem. Have several students sketch their diagrams on the board and list their results. Elicit that although the triangles have different sized angles, and are different shapes, the sum of the measures of the angles seem always to be about 180^{o}. Demonstrate, by using a large triangle cut from a piece of paper, that if the angles of the triangle are placed next to each other, it appears that a straight angle is formed. (See diagram.) Recall that the measure of a straight angle is 180^{o}.

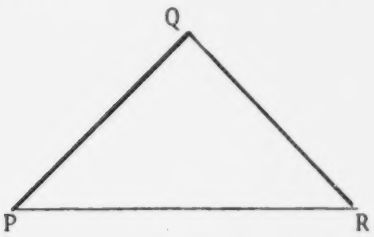




Conclude that the sum of the measures of the angles of any triangle is 180^{o}. (Note: The teacher should emphasize that, in this case, the students used an approximation (measurement) to demonstrate a property of triangles. This is <u>not</u> a mathematical proof.)

2. Do application A.

3. Apply the relationship to find the missing angle of a triangle.



Example: In the figure, the measure of $\angle Q$ is 97^{o} and the measure of $\angle R$ is 38^{o}. Find the measure of $\angle P$

Solution: The sum of the measures of all three angles is 180^{o}.

$$\begin{array}{r} \angle Q + \angle R = 97^{o} \\ + \ 38^{o} \\ \hline 135^{o} \end{array}$$

The measure of the remaining angle, $\angle P$, must be $\begin{array}{r} 180^{o} \\ - \ 135^{o} \\ \hline 45^{o} \end{array}$

The measure of $\angle P = 45^{o}$.

4. Do application B.

5. Have the students attempt to draw a triangle with two right angles. Discuss why it is impossible to do this problem. In a similar manner, lead students to conclude that it is impossible for a triangle to have two obtuse angles or one right and one obtuse angle.

APPLICATIONS:

A. Determine whether each of the following sets of angles can be the three angles of a triangle:

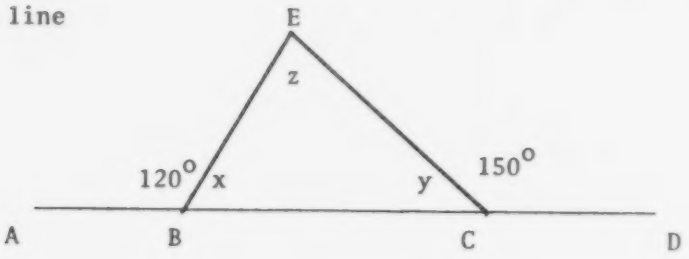
1. 80^{o}, 60^{o}, 40^{o}
2. 42^{o}, 48^{o}, 100^{o}
3. 90^{o}, 90^{o}, 10^{o}
4. 50^{o}, 60^{o}, 65^{o}
5. $37\frac{1}{2}^{o}$, 120^{o}, $22\frac{1}{2}^{o}$
6. 90^{o}, 110^{o}, 15^{o}

B. Find the measure of the third angle of a triangle if the first two angles contain:

1. 70^{o}, 30^{o}
2. 130^{o}, 40^{o}
3. 60^{o}, 60^{o}
4. 90^{o}, 50^{o}
5. $25\frac{1}{2}^{o}$, $47\frac{1}{4}^{o}$
6. 60^{o}, 95^{o}

SUMMARY:

ABCD is a straight line



Find the measure of each of the angles x, y, and z. Have students determine the total number of right angles, obtuse angles, straight angles or acute angles that may be found in any one triangle.

LESSON 4

AIM: To classify triangles according to the measures of their angles

PERFORMANCE OBJECTIVES: The student will be able to:

- determine when a triangle is acute, right, or obtuse.
- state that equilateral triangles are equiangular.
- state that an isosceles triangle has two angles of equal measure.
- name the parts of a right triangle.
- state that the hypotenuse is the longest side of a right triangle, and is opposite the right angle.
- state that the legs of a right triangle form the right angle.

VOCABULARY:

New Terms: acute triangle, right triangle, obtuse triangle, equiangular, leg, hypotenuse, vertex angle, base angles

CHALLENGE PROBLEM:

Draw an isosceles triangle containing one right angle.

DEVELOPMENT:

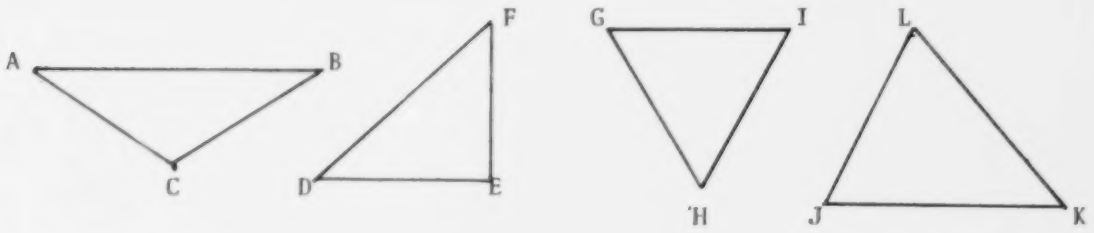
1. Draw and classify triangles by angles: "acute," "right," and "obtuse."
2. Allow pupils to explore such questions as the following:
   a. Can a triangle be both scalene and acute? Scalene and right? Scalene and obtuse?
   b. Can an isosceles triangle be acute? Right? Obtuse?
   c. Can a right triangle be equilateral? Can an obtuse triangle be equilateral?

   (Note: Teachers should prepare a rexograph sheet and large cut-out models illustrating each type of figure.)
3. Help pupils to discover the concepts that an equilateral triangle is equiangular and that an isosceles triangle has two angles of equal measure. Use models made of construction paper or use extensible figures. Define "vertex angle" and "base angles."

4. Define the terms "leg" and "hypotenuse" for the sides of the right triangle and help pupils to discover that the hypotenuse is the longest side of the right triangle, that it lies opposite the right angle, and that the legs form the right angle.

APPLICATION:

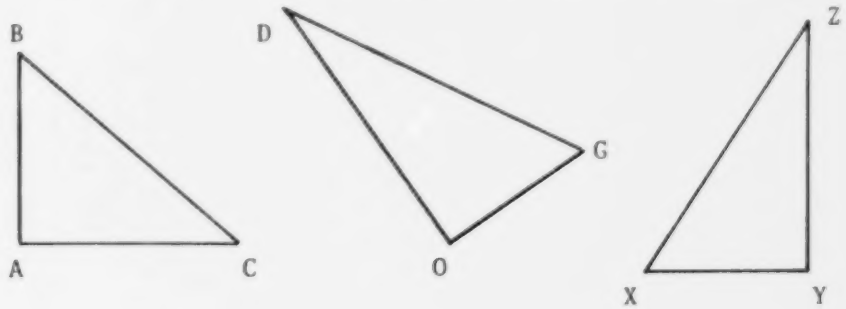
A. Classify the following triangles according to angles.



B. Classify the triangles in application A according to their sides.

C. 1. Determine, by measuring, which triangle is a right triangle.

2. Name the legs and the hypotenuse of each right triangle.



SUMMARY:

1. Elicit a review of the new vocabulary introduced in this lesson.
2. Test the pupils' understanding using such questions as:
   (a) Are all equilateral triangles equiangular?
   (b) Is an isosceles triangle equiangular?
   (c) Can a triangle be drawn having one right angle and one obtuse angle? Two right angles? Two obtuse angles?

CLIFF-HANGER:

Using a metric ruler, draw a right triangle with one leg 5 cm and one leg 12 cm. Find the length of the hypotenuse in centimeters.

LESSON 5

AIM: To study the Pythagorean relationship

PERFORMANCE OBJECTIVES: The student will be able to:

- state that a and b in the Pythagorean relationship ($a^2 + b^2 = c^2$) are the legs, and c is the hypotenuse.
- make a list of squares of whole numbers from 1 to 20.
- express a perfect square as the second power of one of its factors.
- state that a perfect square can be written as the product of two equal factors.
- determine if a triangle is a right triangle using the Pythagorean relationship.

VOCABULARY:

New Terms: Pythagorean relationship, perfect square

Review: hypotenuse, legs, right triangle

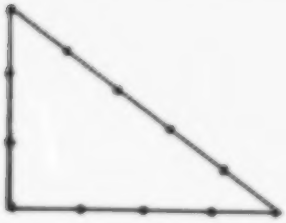
CHALLENGE PROBLEM:

Using an inch ruler, draw a right triangle whose legs are 3" and 4". Find the length of the hypotenuse (without measuring).

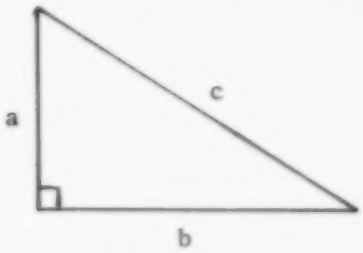
DEVELOPMENT:

1. Pupils should realize that finding the answer to the challenge problem depends on being able to calculate the length of the hypotenuse of a right triangle when the lengths of its legs are known (in this case, 3 and 4). Have the class determine the answer to this problem by measuring the length of the hypotenuse on a carefully drawn scale drawing. Review the cliff-hanger from the previous lesson for a right triangle whose legs are 5 and 12. Although pupils will be able to obtain answers for the length of the hypotenuse in each case, they should realize that the direct measurement process may be inaccurate.
2. Tell the story of the Egyptian rope stretchers who reportedly used a rope with twelve evenly spaced knots to assure that the angles

at the corners of the pyramids were truly right angles. Pupils should conclude from a demonstration that a triangle whose sides measure 3, 4, and 5 must be a right triangle. If time permits, this may also be shown experimentally for a 5, 12, 13 right triangle.



3. State the Pythagorean relationship for right triangles whose legs are a and b, and whose hypotenuse is c. It may be expressed in the form of a formula: $a^2 + b^2 = c^2$



The Pythagorean relationship may be expressed in words as follows: The sum of the squares of the legs equals the square of the hypotenuse.

The students may be told that the Pythagorean relationship was named for Pythagoras, a Greek mathematician who lived during the sixth century B.C.

4. Review the meaning of a^2 as "a X a" which may be read "a to the second power" or "a squared."
5. Do application A.
6. Inform students that each answer in application A is called a perfect square. Define a <u>perfect square</u> as the second power of one of its factors or the product of two equal factors.

7. Demonstrate the use of the Pythagorean relationship for the following problem. "If a triangle has sides of 8 m, 17 m, and 15 m, determine whether or not it is a right triangle." Review that the hypotenuse is the longest side in a right triangle; therefore the largest of these numbers will be substituted for c in the formula. The remaining numbers may be substituted for either a or b.

$c = 17, \quad a = 8, \quad b = 15$

$a^2 + b^2 = c^2$

$64 + 225 = 289$

$289 = 289$

Since $a^2 + b^2 = c^2$, the triangle is a right triangle.

8. Do application B.

APPLICATIONS:

A. Make a table of the squares of the numbers 1 thru 20.

number	2nd power	two equal factors	perfect square
1	1^2	1×1	1
.	.	.	.
.	.	.	.
.	.	.	.
20	20^2	20×20	400

B. Tell whether the measurements in each group can be the lengths of the sides of a right triangle:

1. 6 ft, 8 ft, 10 ft
2. 10 cm, 15 cm, 20 cm
3. 7", 4", 5"
4. 7 m, 24 m, 25 m
5. 16 mm, 20 mm, 12 mm

SUMMARY:

Review the algebraic representation of the Pythagorean relationship and have students explain how it can be used to determine whether or not a triangle is a right triangle.

CLIFF-HANGER:

In using the Pythagorean relationship, a student obtained the result 676 as the square of the hypotenuse. What is the length of the hypotenuse?

LESSON 6

AIM: To find square roots of perfect squares and apply the Pythagorean relationship

PERFORMANCE OBJECTIVES: Students will be able to

- read the symbol $\sqrt{}$ as "the square root of"
- state that the square root of a number is one of its two equal factors.
- find the square roots of perfect squares.
- use the Pythagorean relationship and the concept of finding a square root (by trial and error) to find the hypotenuse of several right triangles with whole numbered sides.
- find the length of a leg of a right triangle using the Pythagorean relationship.

VOCABULARY:

New Terms: equal factors, square roots

Review: Pythagorean relationship

CHALLENGE PROBLEM:

If the square of the hypotenuse of a right triangle is 121, what is the length of the hypotenuse?

DEVELOPMENT:

1. Discuss challenge problem. Review definition of perfect square. Write the statement 121 equals 11^2 or 11 X 11. Elicit that the hypotenuse is 11. Name 11 as the square root of 121. Define square root of a number as one of its two equal factors. Introduce the symbol $\sqrt{}$ as meaning "the square root of"
2. Do applications A and B.
3. Find the hypotenuse of a right triangle the measures of whose legs are 10 and 24. Have pupils write the formula, substitute, and compute the squares.

$$a^2 + b^2 = c^2$$

$$10^2 + 24^2 = c^2$$

$$100 + 576 = c^2$$

$$676 = c^2$$

The students should be led to recall the cliff-hanger from the previous lesson. Solving, we have $c = 26$ because $26 \times 26 = 676$.

4. In right triangle ABC, hypotenuse $\overline{AB}$ measures 10" and leg $\overline{BC}$ measures 8". Use the Pythagorean relationship to find the length of $\overline{AC}$.

   Solution: $(AC)^2 + (BC)^2 = (AB)^2$

   $(AC)^2 + 8^2 = 10^2$

   $(AC)^2 + 64 = 100$

   By trial and error students should discover that $(AC)^2$ must equal 36, hence $\overline{AC} = 6$.

5. Do applications C and D.

APPLICATIONS:

A. Complete each statement.

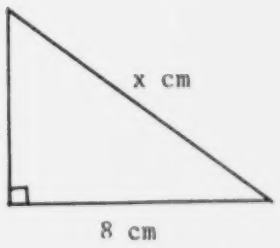
1. The square root of 16 is _____
2. $\sqrt{49}$ = _____
3. 8^2 = _____
4. The square of 9 is _____
5. $\sqrt{\quad}$ = 10
6. _____ squared = 400

B. Find the positive value of the variable:

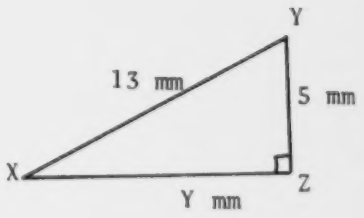
1. If $x^2 = 81$ then x =
2. If $y^2 = 225$ then y =
3. If $100 = m^2$ then m =
4. If $24 = p^2$ then p = (ans. p = $\sqrt{24}$)

C. Find the missing length in each right triangle.

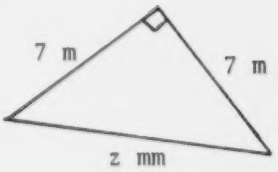
1.

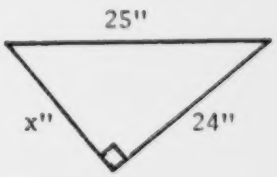


2.



3.

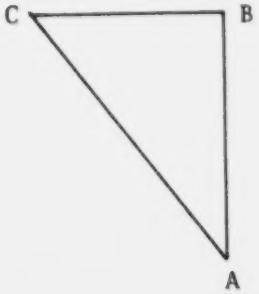




D. A ladder 25 feet long is placed against the side of a building so that the bottom of the ladder is on the ground seven feet from the base of the building. How far up the side of the building will the top of the ladder reach?

SUMMARY:

Review vocabulary words. Present the problem: Three markers in a lake, A, B, and C are situated so that ∠ABC is a right angle.



AB is 40 meters and BC is 30 meters. José swam from A to B and from B to C. Larry swam directly from A to C.

How much further did José swim?

LESSON 7

AIM: To identify quadrilaterals and to distinguish among the various special quadrilaterals, including the parallelogram, rectangle, rhombus, square, and trapezoid

PERFORMANCE OBJECTIVES: The student will be able to:

- define quadrilateral
- recognize the "special" quadrilaterals by their shapes.
- use the terms equilateral and equiangular when referring to the appropriate quadrilaterals.
- recognize the differences (and similarities) among the various types of quadrilaterals, emphasizing parallel lines, lengths of sides, and right angles.
- know some of the properties of the diagonals in the quadrilaterals (esp.: bisecting each other, equal lengths, and forming right angles).

VOCABULARY:

New Terms: quadrilateral, parallel, parallelogram, rectangle, rhombus, square, trapezoid, diagonal

CHALLENGE PROBLEM:

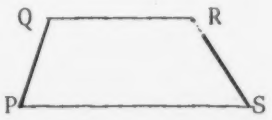
How many different kinds of four-sided closed figures can you draw?

DEVELOPMENT:

1. Have many students display their diagrams on the board. Define the word "quadrilateral" as a closed figure having four sides.
2. Discuss the concept of "parallel lines"; show the symbolic representation ($AB \parallel CD$), and indicate some pairs of parallel lines using the diagrams as models.

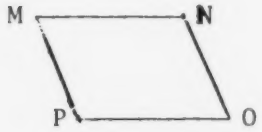
3. Select the quadrilaterals: (a) square, (b) rectangle, (c) parallelogram, (d) rhombus, and (e) trapezoid, from among the figures drawn, and list some distinguishing characteristics of each.

Example 1.



Trapezoid PQRS where $\overline{QR} \parallel \overline{PS}$

Example 2.



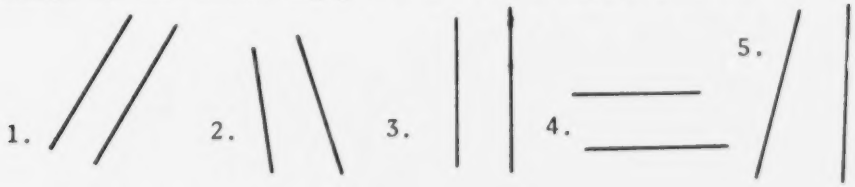
Parallelogram MNOP
$MN \parallel PO$, $\overline{MN} = \overline{PO}$
$NO \parallel MP$, $\overline{NO} = \overline{MP}$

4. Discuss the differences among the various types of quadrilaterals.
   (a) Is every square a parallelogram?
   (b) Is every square a rectangle?
   (c) Is every rectangle a square?
   (d) Is every rhombus a parallelogram?
   (e) Can a trapezoid also be a parallelogram?
   (f) Can a trapezoid be equilateral?
   (g) Which parallelogram is equilateral? Which is equiangular? Which is both equilateral and equiangular?
   (h) Discuss the diagonals of selected quadrilaterals.

5. Do applications.

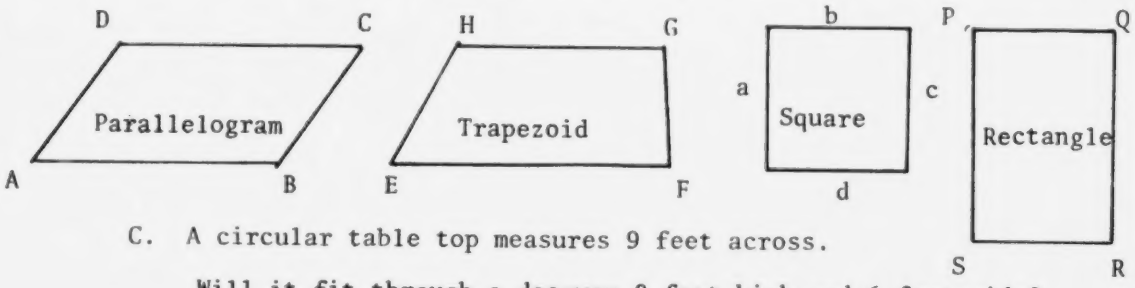
APPLICATIONS:

A. Which of the following pairs of lines appear parallel?



B. 1. In each diagram below, name the parallel sides in each figure.

2. Name the sides of equal measure in each figure



C. A circular table top measures 9 feet across.

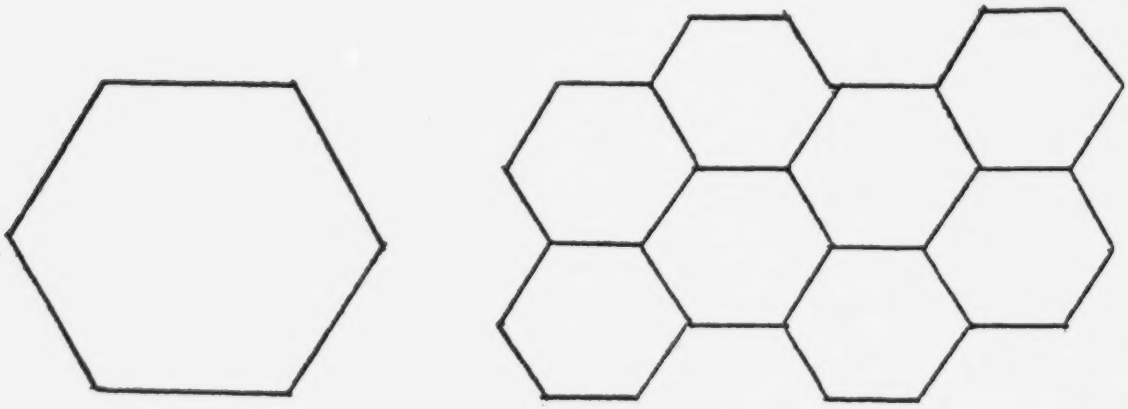
Will it fit through a doorway 8 feet high and 6 feet wide?

SUMMARY:

Elicit a review of the new vocabulary introduced in this lesson.

CLIFF HANGER:

Bees build honeycombs in the shape shown. What is this shape called?



LESSON 8

AIM: To identify polygons of more than four sides

PERFORMANCE OBJECTIVES: The student will be able to:

- name polygons containing 5, 6, 8, 10, and 12 sides.
- define "regular polygon."
- state the relationship among the angles of some polygons that allows them to tessellate or "fit" together. (The sum of the measures of the angles about a point = 360^{o}.)
- state a relationship between the number of sides and the number of diagonals in a polygon.
- state a relationship between the number of sides in a polygon and the number of triangles with a common vertex that may be found in the polygon.

VOCABULARY:

New Terms: polygon, pentagon, hexagon, octagon, decagon, dodecagon, regular, vertices

CHALLENGE PROBLEM:

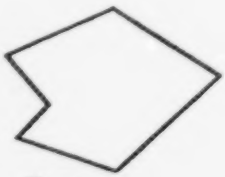
Refer to cliff-hanger in previous lesson.

DEVELOPMENT:

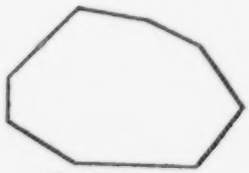
1. Identify and illustrate each of the following terms, placing emphasis on the prefixes in the names of each of the polygons: (a) pentagon (five sides, e.g., Department of Defense building in Washington, D.C.), (b) hexagon (six sides, e.g., honeycomb, floor tiles, etc.), (c) octagon (eight sides, e.g., stop signs, floor tiles, octave in music), (d) decagon (ten sides, e.g., decade, decimal) (e) dodecagon (twelve sides, e.g., dodecaphonic)
2. Define "regular" polygons as polygons that are both equilateral and equiangular, and show by the use of diagrams that each of the figures mentioned above may or may not be regular. (Note: In

quadrilaterals, a rhombus is equilateral, a rectangle is equiangular, but a square is regular - See application A, 2.)

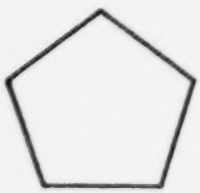
Pentagon
(5 sides)



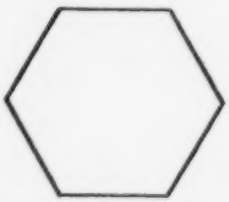
Hexagon
(6 sides)



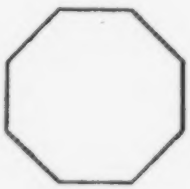
Octagon
(8 sides)



Regular Pentagon



Regular Hexagon

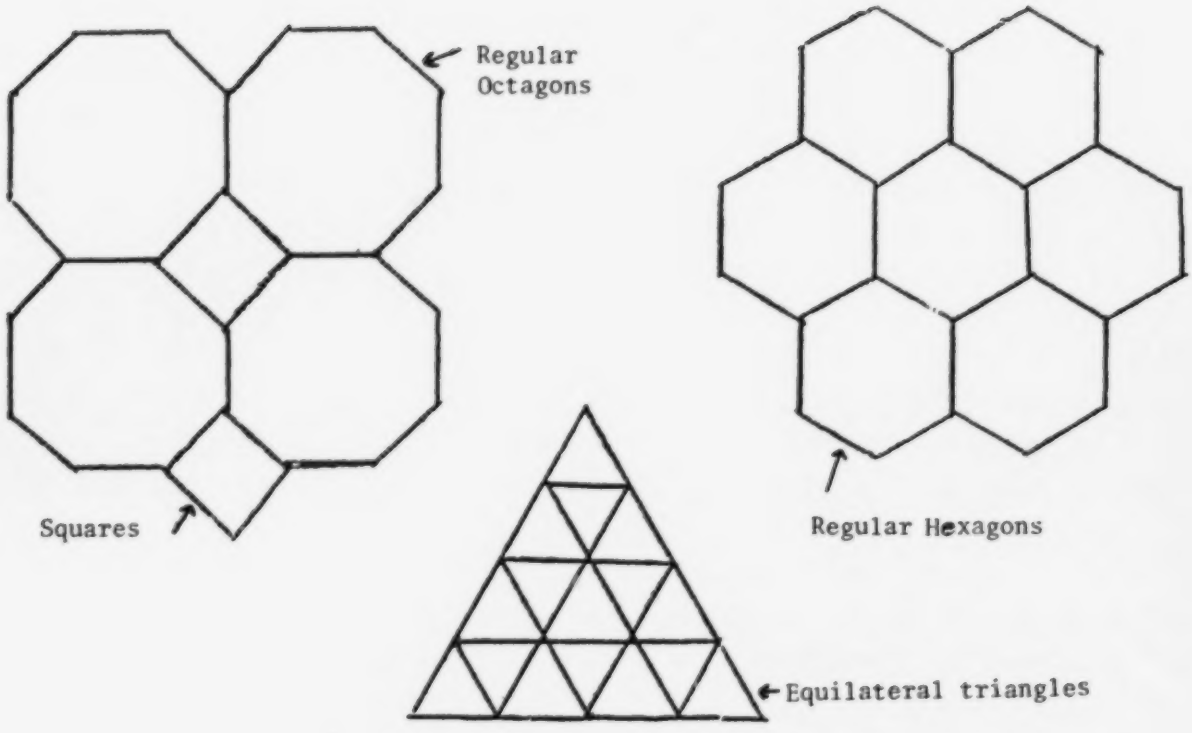


Regular Octagon

3. Do application A.

4. Teacher may choose to include either or both of the following in class discussion:

   Designs, such as those used in floor tiles, linoleum, and wallpaper, may be created by the pupils using patterns of one or more of the regular polygons, as shown in the accompanying diagrams.



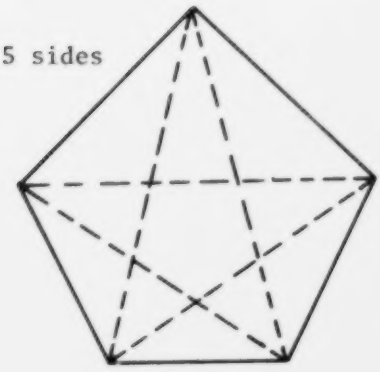
2. As time and pupil interest permit:

   (a) Consider the measures of the interior angles of some of the polygons mentioned above. This will help pupils determine why some regular polygons (such as equilateral triangles, squares, and regular hexagons) can be used in designs. They "fit" together. Others, such as regular pentagons, do not fit together. Still others, such as regular octagons with squares, as shown above, can only be used in special combinations.

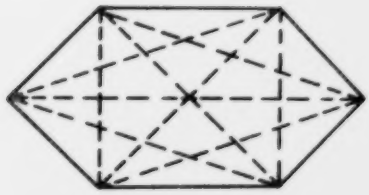
   (b) Study the comparison between the number of sides and the number of diagonals of a polygon. Note to teacher: A polygon of n sides has $n\ C_2 - n$ or $\frac{n\ (n-1)}{2} - n$ diagonals. Have students complete a table similar to the one shown:

Number of Sides	Number of Diagonals
3	0
4	2
5	5
6	9

5 sides



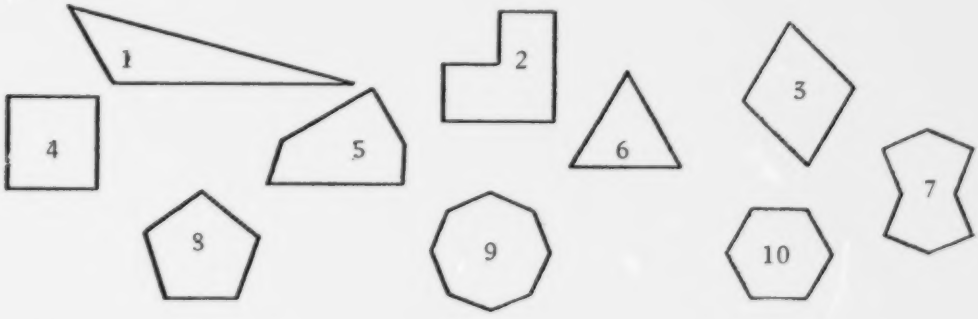
6 sides



(c) Divide polygons into triangles by selecting one vertex of each polygon and drawing all possible diagonals from that vertex. Pupils should discover that a polygon of n sides may be divided into n-2 triangles by this method.

APPLICATIONS:

A. Use these figures to answer the following questions.



1. a. Which figures are triangles?
   b. A regular triangle is called an equilateral triangle. Which figure is an equilateral triangle?
2. a. Which figures are quadrilaterals.
   b. Are all the sides of each quadrilateral equal in measure?
   c. Are both quadrilaterals regular quadrilaterals?
   d. What is a regular quadrilateral called?
3. Which figures are pentagons? Which figure is a regular pentagon?
4. Which figure is a regular hexagon? Which figure is a regular octagon?

SUMMARY:

Elicit a review of the new vocabulary introduced in this lesson.

LESSON 9

AIM: To identify and discuss some of the points and lines in a circle

PERFORMANCE OBJECTIVES: The student will be able to:

- define a circle as the figure formed by all the points a given distance from a fixed point.
- state that a circle can be approximated by constructing a regular polygon with a "large" number of sides (24 sides, for example).
- define and draw a radius, a diameter, a chord, an arc, a sector.
- state the relationship between a radius and a diameter.
- solve arithmetic problems relating diameters and radii, which use fractions and decimals.
- construct a circle using a compass.

VOCABULARY:

New Terms: circle, center, radius, diameter, chord, arc, sector, compass

CHALLENGE PROBLEM:

A dog is tied to a pole by a leash. It walks around the pole, always pulling tightly on the leash. What path does it follow as it walks?

DEVELOPMENT:

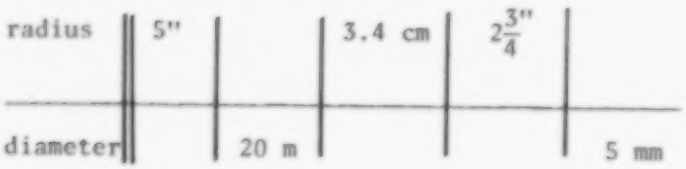
1. Discuss with the pupils the concept of the circle as a path of all points equidistant from a given point, called the "center," as illustrated by the dog on the leash. Include other illustrations such as treasure hunting (a map shows a buried treasure on an island 100 yards from the only tree on the island, etc.)
2. Discuss informally the concept of the circle as the limit of a regular polygon as the number of sides increases beyond all bounds.
3. Define and illustrate each of the following terms:
   (a) center (b) radius (c) diameter (d) chord (e) arc of a circle (f) sector (slice of pizza)
4. Explore special relationships among lines of the circle defined

above. For example, (a) the length of a radius is equal to one-half the length of the diameter, or the length of the diameter is twice the length of the radius, (b) all radii of a circle are equal in length, and all diameters of a circle are equal in length, (c) the diameter is the longest chord in a circle.

5. Demonstrate use of compass.

APPLICATIONS:

A. Complete the following table of values:



B. 1. Draw a circle whose radius is 10 cm.

2. In this circle draw chord AB = 12 cm.

3. In this circle draw chord CD = 20 cm. What is another name for $\overline{CD}$?

SUMMARY:

Elicit a review of the new vocabulary and formulas for this lesson.

LESSON 10

AIM: To calculate the perimeter of any polygon

PERFORMANCE OBJECTIVES: The student will be able to:

- define perimeter.
- use fractions and decimal numbers to calculate perimeters of polygons.

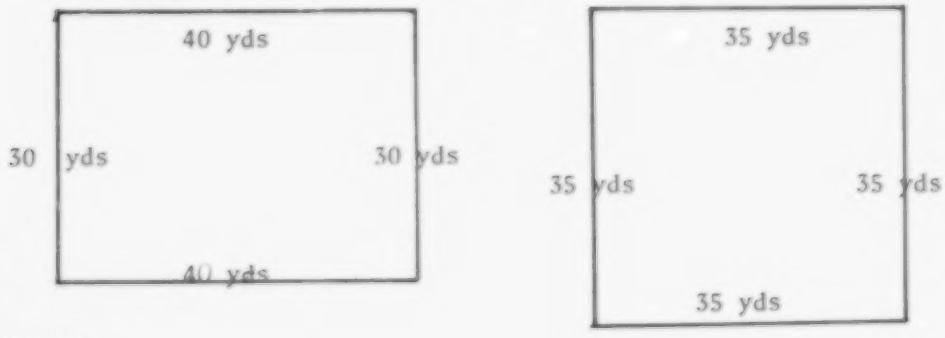
VOCABULARY:

New Term: perimeter

Review: polygon, rectangle, square

CHALLENGE PROBLEM:

As shown in the diagram below, a rectangular lot has a length of 40 yards and a width of 30 yards. A square lot has a side of 35 yards. How does the amount of fencing needed to enclose the rectangular lot compare to the amount of fencing needed for the square lot?

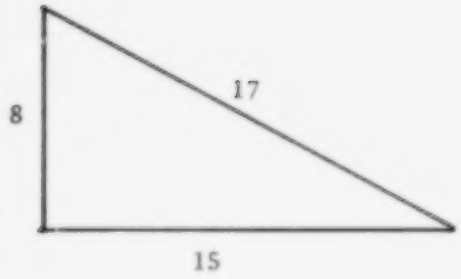


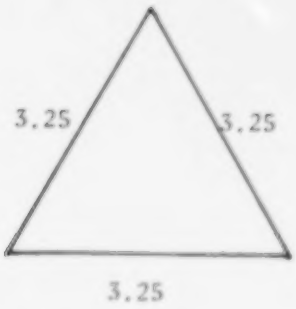
DEVELOPMENT:

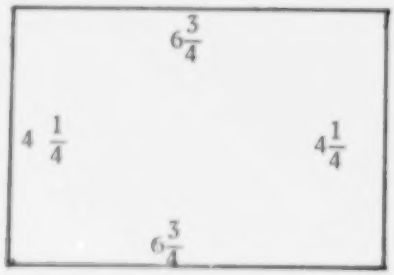
1. Using the challenge problem, define the term "perimeter." Other types of polygons should be illustrated and their perimeters calculated. Exercises should involve decimals and fractions with like and unlike denominators. Typical exercises are indicated below. The teacher should supply additional exercises of graduated difficulty. (It will be necessary to spend a major portion of this lesson on addition of decimals.)

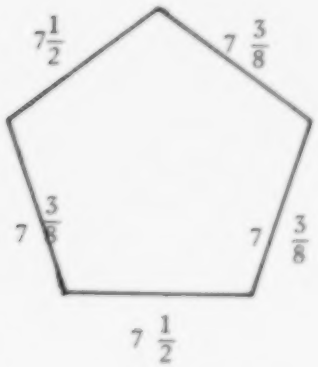
APPLICATIONS:

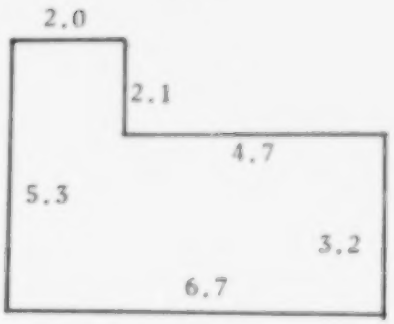
1. Calculate the perimeter of each of the following figures:

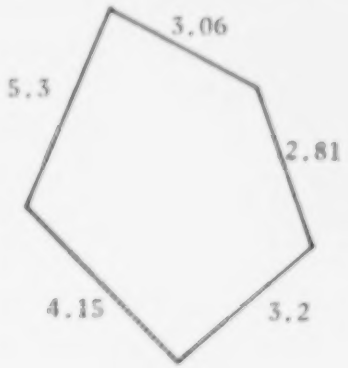












2. Include several verbal applications. Emphasize key words or phrases that indicate perimeter.

   (a) Decorative molding is being placed around a window. If the window is rectangular and measures 3 feet across the top and bottom and 5 feet on each side, how many feet of molding is needed to go around the window?

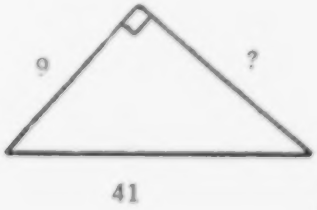
   (b) How many meters of fencing is needed to enclose a square piece of land 40.5 meters on each side?

SUMMARY:

Review the meaning of the perimeter of a polygon.

CLIFF-HANGER:

As shown in the diagram, the perimeter of a right triangle is 90, one leg is 9 and the hypotenuse is 41. Find the length of the other leg.



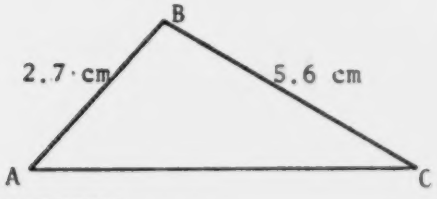
LESSON 11

AIM: To solve various application problems involving perimeters of polygons

PERFORMANCE OBJECTIVES: The student will be able to:

- use some properties of triangles (isosceles, equilateral) and polygons when solving perimeter problems.
- find the length of a missing side of a polygon when the perimeter and the other sides are given.
- perform subtractions with mixed numbers.

CHALLENGE PROBLEM:



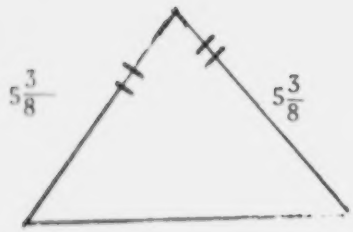
If the perimeter of triangle ABC is 14 cm, find the length of side AC.

DEVELOPMENT:

1. Discuss challenge problem and relate to cliff-hanger of the previous lesson. Elicit how the missing side of a polygon may be found when the lengths of the other sides and the perimeter are known. The teacher should provide other exercises of a similar type, involving subtractions as well as additions of whole numbers, fractions, and decimals. Typical exercises are indicated. The teacher should supply additional ones of graduated difficulty.

2. Do application A.

3. Discuss the following problem:
   One of the equal sides of an isosceles triangle has a length of $5\frac{3}{8}$ and the perimeter of the triangle is 14. Find the length of the third side of this triangle (called the "base"). Draw a diagram and determine a notation for showing 2 sides are equal on this diagram.



Use marks on the legs to indicate equality.

Solution:
$$\begin{array}{r} 5\frac{3}{8} \\ +\ 5\frac{3}{8} \\ \hline 10\frac{6}{8} \end{array}$$

Exchange:

$$14 = 13 + 1 = 13\frac{8}{8}$$

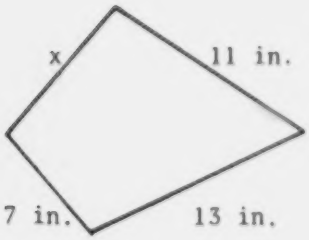
$$\begin{array}{rr} -\ 10\frac{6}{8} & -\ 10\frac{6}{8} \\ \hline & 3\frac{2}{8} = 3\frac{1}{4} \end{array}$$

It is necessary to review subtraction of mixed numbers with exchanges. (See lesson #13 in Unit II, page 58.)

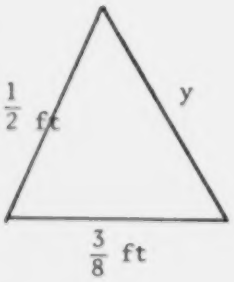
4. Do applications B and C.

APPLICATIONS:

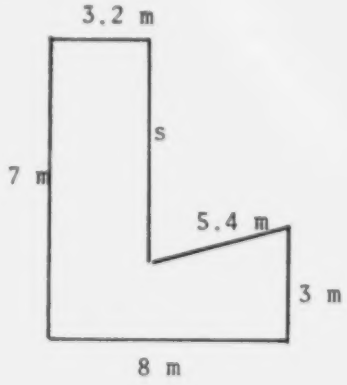
A. Find the length of the missing side in each diagram.



p = 39 in.



P = $\frac{3}{2}$ ft



P = 32.5 m

B. Find the differences:

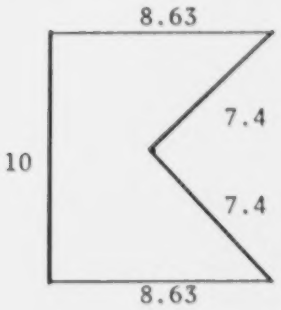
1. $4\frac{1}{3} - 1\frac{2}{3}$
2. $12 - 6\frac{1}{4}$
3. $25 - 6\frac{7}{8}$
4. $8\frac{1}{4} - 3\frac{1}{2}$
5. $15\frac{1}{3} - 6\frac{3}{4}$

C. 1. The perimeter of a regular hexagon is 30. Find the length of one side.

2. The perimeter of a regular pentagon is 27.5. Find the length of one side. (Review division of decimals by whole numbers.)

3. The length of one side of an equilateral triangle is $7\frac{1}{3}$. Find the perimeter of the triangle (by addition).

4. Find the perimeter of the polygon shown.



SUMMARY:

Review the method of finding the perimeter or any side of a polygon, as studied in this lesson.

CLIFF-HANGER:

An ice skating rink is in the shape of a circle. How would you find its circumference?

LESSON 12

AIM: To calculate the circumference of a circle using $\pi = 3.14$

PERFORMANCE OBJECTIVES: Students will be able to:

- define circumference.
- measure circumferences of common objects using string and a ruler, or a tape measure.
- find an approximate value of π by relating the circumference and the diameter of objects.
- find the circumferences of circles using 3.14 for π, where the radii and diameters are decimals or whole numbers.

VOCABULARY:

New Terms: circumference, pi (π)

Review: radius, diameter, circle

CHALLENGE PROBLEM:

An ice skating rink is in the shape of a circle. How would you find the circumference?

DEVELOPMENT:

1. Review the meaning of the words "radius" and "diameter," and define "circumference" as a word meaning the perimeter of a circle. Discuss the use of a rolling marker to measure the circumference of a circular ice skating rink. Allow students to measure approximately, with ropes, strings, tape measure, or rulers, the radius, diameter, and circumference of a number of circles, including such common objects found in the classroom as the waste basket, circular containers (pencil boxes, chalk cans, etc.), etc., as well as prepared circles drawn on a paper or cut from cardboard. Several types of experiments may be performed. Elicit as many relationships as possible concerning the circumference, radius, and diameter of each circle. Some of these will be:

a. the diameter of a circle is always twice the radius

b. the radius of a circle is always one-half the diameter

c. the circumference of a circle is always a little more than three times the diameter: C = d X (a little more than 3). The teacher may then replace "a little more than 3" with the symbol π (read "pi"). An approximation of π is 3.14. Write the formula $C = \pi d$.

2. Follow up on the challenge problem by asking: If the diameter of the circular ice skating rink is 150 feet, find the length of the railing that surrounds the rink.

Procedure:

(1) Write the formula $C = \pi d$

Substitute values $\pi = 3.14$, $d = 150$

$C = 3.14 \times 150$

Review multiplication of decimals with particular emphasis on the placement of the decimal point.

$$\begin{array}{r} 150 \\ \times\ 3.14 \\ \hline 600 \\ 150 \\ 450 \\ \hline 471.00 \end{array}$$

The length of the railing is 471 feet.

3. Do applications.

APPLICATIONS:

Using the approximation of 3.14 for the value of π, calculate the circumference of a circle with the given dimensions.

1. diameter = 10 inches
2. diameter = 7.5 cm
3. radius - 15 km (note need for first finding the diamter of this circle, or you may introduce the formula $C = 2\pi r$)
4. radius = 2.4 m

SUMMARY:

1. Review the new vocabulary terms. Elicit one commonly used approximation of π (3.14).
2. Calculate the circumference of a circle whose diameter is 3.5 using the formula c =

LESSON 13

AIM: To calculate the circumference of a circle using $\pi = \frac{22}{7}$

PERFORMANCE OBJECTIVES: Students will be able to:

- find the circumferences of circles using $\frac{22}{7}$ for π where the radii and diameters may contain fractions

VOCABULARY:

Review: circumference, pi (π), radius, diameter

CHALLENGE PROBLEM:

Is the circumference of a circle whose diameter is $3\frac{1}{2}$" greater than, less than, or equal to the perimeter of a square whose side measures 2.8"

DEVELOPMENT:

1. Discuss challenge problem. Elicit the fact that substitution in the formula yields $C = 3\frac{1}{2} \times 3.14$ which is an awkward computation to perform. Introduce another approximation, $\pi = \frac{22}{7}$. Now $C = 3\frac{1}{2} \times \frac{22}{7}$

$$C = \frac{1}{2}\cancel{7}_{1} \times \frac{11}{\cancel{22}}\cancel{7}_{1}$$

$$C = 11"$$

Also, $P = 4(2.8)$

$$P = 11.2"$$

Thus: The circumference of the circle is less than the perimeter of the square.

2. This solution should lead to a discussion of multiplying fractions and mixed numbers. Instruct students that when multiplying mixed numbers they must first change them to improper fractions. Practice exercises in multiplication of fractions should be provided.
3. Do Application A.
4. Compare the result of the challenge problem using $\pi = \frac{22}{7}$ with using $\pi = 3.14$.

$$C = 3.5 \times 3.14$$

$$C = 10.99"$$

5. Do Application B.

APPLICATIONS:

A. Multiply

1. $\frac{3}{4} \times \frac{1}{2}$
2. $\frac{5}{8} \times \frac{4}{9}$
3. $1\frac{1}{2} \times \frac{3}{4}$
4. $\frac{1}{4} \times 3\frac{1}{2}$
5. $2\frac{1}{2} \times 3\frac{1}{3}$
6. $6 \times 4\frac{2}{3}$
7. $\frac{1}{3} \times 3\frac{1}{4}$
8. $2\frac{15}{16} \times 1\frac{1}{7}$

B. Using the approximation $\frac{22}{7}$ for the value of π, calculate the circumference of a circle with a given dimension.

1. diameter = $\frac{5}{8}$ inches
2. diameter = $12\frac{1}{4}$ feet
3. radius = $3\frac{1}{2}$ inches
4. radius = $9\frac{1}{3}$ inches

SUMMARY:

Review the two approximations to π (pi) introduced in the past two lessons. Discuss when each might be used. Review the formulas used in calculating the circumference of a circle.

LESSON 14

AIM: To compute perimeters and circumferences

PERFORMANCE OBJECTIVES: Students will be able to:

- determine which method of calculating the distance around a figure is required to solve a problem
- compare perimeters and circumferences of various polygons and circles

VOCABULARY:

Review: circumference, perimeter, and all terms relating to polygons and circles

CHALLENGE PROBLEM:

Two pieces of land must have fences built around them. One is in the shape of an equilateral triangle with each side measuring 80 feet. The other is in the shape of a square with each side measuring 70 feet. Which piece of land needs more fencing? How much more?

DEVELOPMENT:

This lesson consists entirely of practice exercises in perimeters of polygons and circumferences of circles. Begin the discussion with the challenge problem and continue with the problems that follow. The teacher should supply additional exercises, as time permits, of graduated difficulty, including some with decimals and fractions.

APPLICATIONS:

A. Find the perimeter of each of the following rectangular lots:

	length	width
1.	13.5 mm	6.2 mm
2.	13.25 cm	4.70 cm
3.	8.2 ft	7.25 ft

	length	width
4.	$6\frac{1}{2}$ mm	$7\frac{1}{2}$ mm
5.	$8\frac{1}{3}$ m	$10\frac{2}{3}$ m
6.	$10\frac{1}{2}$ ft	$12\frac{1}{4}$ ft

B. Which is larger, the perimeter of a square whose side is 9 or the circumference of a circle whose diameter is 10?

C. A dog is tied to a pole by a leash 7 feet long. How many feet does it walk around the pole once, always keeping the leash tight? How many feet does it walk if it walks around the pole 3 times, always keeping the leash tight?

D. Fencing costs $4.50 per yard. Two fields are going to be enclosed by a fence. One field is in the shape of an equilateral triangle whose side is 50 feet long. The other is in the shape of a circle whose diameter is 60 feet long. Which field will cost more to enclose?

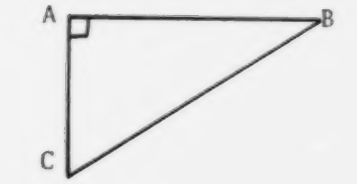
SUMMARY:

Review the meaning of perimeter and circumference and the method used for finding each.

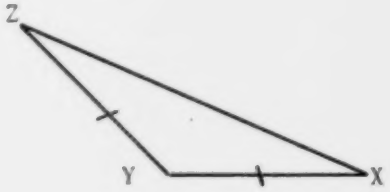
SUGGESTED TEST ITEMS

1. Choose one word from column A and one word from column B to classify, or describe, each of the two triangles pictured. Write this word in the blank provided under each triangle.

COLUMN A	COLUMN B
acute	scalene
right	isosceles
obtuse	equilateral

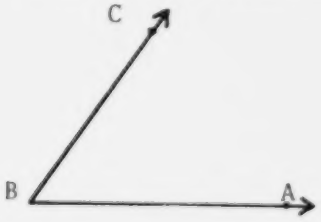
a. 

A _____ B _____

b. 

A _____ B _____

* 2. What is the best approximation for the measure of $\angle ABC$?

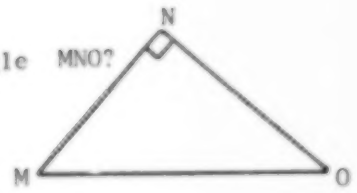


a) 30° b) 60° c) 90° d) 120°

* 3. If a triangle has angles whose measures are 90° and 30°, what is the measure of the third angle?

a) 30° b) 60° c) 120° d) 150°

4. Which represents the hypotenuse of right triangle MNO?



a) $\overline{NM}$ b) $\overline{OM}$ c) $\overline{ON}$ d) $\overline{MN}$

5. Which of the following numbers is not a perfect square?

1) 8 3) 25

2) 16 4) 36

6. Which of the following sets of measures can be the sides of a right triangle?

1) 1, 2, 3 3) 3, 4, 5

2) 2, 3, 4 4) 4, 5, 6

7. What is the numerical value of $\sqrt{16}$?

1) 8 3) 256

2) 2 4) 4

8. Which quadrilateral has only one pair of parallel sides?

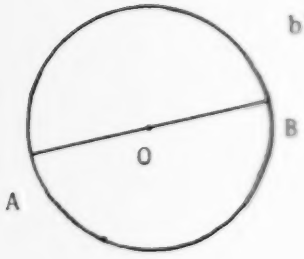
1) square 3) trapezoid

2) rectangle 4) parallelogram

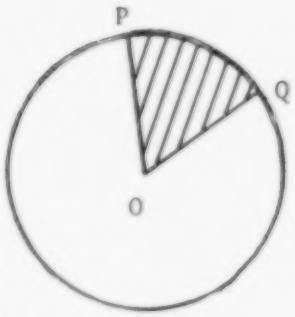
9. Points A and B are on circle O and $\overline{AB}$ passes through the center. $\overline{AB}$ is called a(n)

a) arc c) radius

b) sector d) diameter



10. Points P and Q are on a circle.

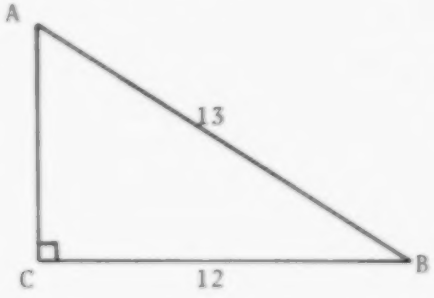


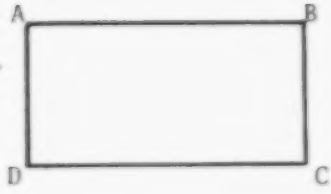
The shaded portion of the circle is called a(n)

a) arc
b) sector
c) radius
d) diameter

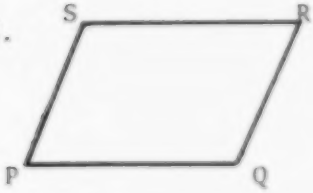
* 11. Using the Pythagorean relationship, what is the length of the hypotenuse of a right triangle whose legs are 6 and 8?

* 12. In the diagram below of right triangle ABC, AB = 13 and BC = 12, using the Pythagorean relationship, what is the length of $\overline{AC}$?



13. 

If ABCD is a rectangle, and AB = 10 cm and BC = 7 cm, what is the length of $\overline{CD}$?

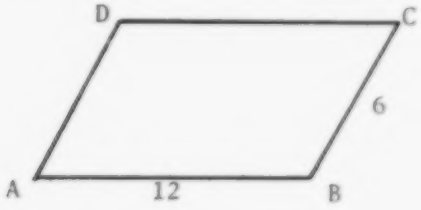
14. 

In parallelogram PQRS, name the side parallel to $\overline{RQ}$?

* 15. What is the perimeter of a triangle whose sides have lengths 5, 6, and 7?

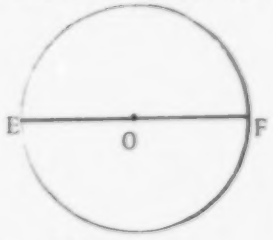
* 16. Each side of a square measures 3.5 centimeters. What is the number of centimeters in the perimeter of the square?

17. What is the perimeter of parallelogram ABCD?



* 18. The circumference of a circle is found by using the formula $C = \pi d$. What is the number of centimeters in the circumference of a circle whose diameter is 14 centimeters? (Use $\pi = \frac{22}{7}$)

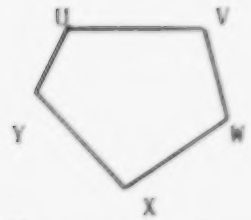
* 19. In the diagram at the right O is the center of the circle. $\overline{EF}$ is drawn through O. If OF = 6, what is the length of a diameter of circle O?



* 20. Using the formula $C = 2\pi r$, what is the circumference of a circle whose radius is 3? (Use $\pi = 3.14$)

21. Choose the word from column II which describes the figure in column I. Place the word in the space provided

I



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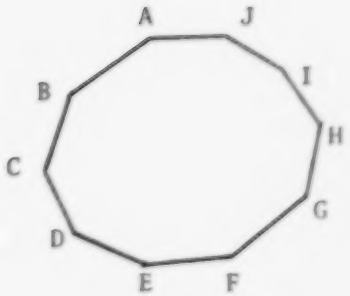

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II

decagon
dodecagon
hexagon
octagon
pentagon

* 22. Add 1.5 + .45 + 7.28

* 23. Subract 1.77
− .85

* 24. Multiply 5.6
× 8.4

* 25. Multiply $\frac{1}{4} \times \frac{3}{5}$

* 26. Subtract $27 - 18\frac{3}{5}$

Unit V. Money Management

LESSON 1

AIM: To learn a method of subtracting dollars and cents from dollar amounts

PERFORMANCE OBJECTIVES: Students will be able to:

- express a monetary value written in cents as a decimal.
- write dollar amounts as dollars and no cents.
- compute the change when items are paid for by $1, $5, $10, or $20 bills.

VOCABULARY:

New Terms: change

CHALLENGE PROBLEM:

Lucie was shopping for her mother. She purchased a loaf of bread for 89¢, a jar of peanut butter for $1.29 and a jar of jelly for 98¢. If her mother gave her a $10 bill, how much change should Lucie receive?

DEVELOPMENT:

1. Discuss challenge problem. Have class recognize a need to express all monetary values as decimals so that students can perform the necessary addition. Elicit that $10 must be expressed as the decimal $10.00. (It will be necessary to spend time reviewing the skill of regrouping when the minuend contains zeros.)

   Solution: Elicit that 89 cents means $.89 or 89 hundredths of a dollar. Similarly, 98¢ = $.98.

   Find the sum of:

   $ .89
   1.29
   .98
   $3.16 cost of 3 items

   To find the change:

   $10.00
   -3.16
   $ 6.84

2. Do applications.

APPLICATIONS: (Assume tax is included in individual prices)

A. Subtract:

$5.00	$10.00	$20.00
− 1.25	− 7.95	− 8.10

3001	8003
− 2511	− 6784

B. Solve the following problems.

1. How much change is given for a $9.92 purchase when a $20 bill is used?
2. Maria had $40 and spent $18.50 for a skirt. How much money did she have left?
3. Find the amount of change from a $10 bill if the following items are purchased:

   1 comb at 79¢

   1 toothbrush at 99¢

   1 toothpaste at $1.19

   1 shampoo at $1.59
4. Find the amount of change from a $20 bill when the following items are purchased in a bakery

   2 chocolate chip cookies at 25¢ each

   6 rolls at 18¢ each

   1 loaf of bread at 98¢

   1 coffee cake at $2.25

SUMMARY:

Review expressing cents as decimal parts of a dollar. Elicit the procedure for finding change. (Note: Teacher may want to use problem 4 in application B as a summary problem.)

LESSON 2

AIM: To learn to use unit pricing in a supermarket

PERFORMANCE OBJECTIVES: Students will be able to:

- state the relationships among liquid measures: ounces; pints; quarts; gallons.
- read and compare samples of supermarket unit price shelf markers to find the best buy.

VOCABULARY:

New Terms: unit price, actual price, pints, quarts, gallons

CHALLENGE PROBLEM:

Three brands of olive oil were being sold in the A & W Supermarket. Brand A was in 1 pint cans and cost $1.79; Brand B was in quart cans and cost $3.29; and Brand C was in gallon cans and cost $8.25. Which brand, A, B, or C,would be considered the "best" buy?

DEVELOPMENT:

1. The relationship to each other of pint, quart, and gallon should be discussed and tabulated on the board.

Units of Liquid Measure

1 pint (pt) = 16 fluid ounces (fl oz)
1 quart (qt) = 2 pints (pt)
1 gallon (gal) = 4 quarts (qt)

2. Compare the costs of the three brands, A, B, and C.

	Pint	Quart	Gallon
Brand A	$1.79	2 X $1.79=$3.58	4 X $3.59 = $14.32
Brand B	--	$3.29	4 X $3.29 = $13.16
Brand C	--	--	$8.25

Thus, Brand C is considered the "best" buy.

3. The teacher should remind the students that this method was used in an earlier lesson. It is no longer needed because of unit pricing. The teacher should use large unit price cards from the supermarket as examples and discuss unit pricing with the class. The unit price cards of Brands A, B, and C would look like this:

UNIT PRICE	YOU PAY
$14.32 per pt	$1.79 1 pt
BRAND A	

UNIT PRICE	YOU PAY
$13.16 per qt	$3.29 1 qt
BRAND B	

UNIT PRICE	YOU PAY
$8.25 per gal	$8.25 1 gal
BRAND C	

4. Emphasize the difference between unit price and actual price. If these three cards were displayed on the shelf of the supermarket, you would need only to look at the unit price to see which has the lowest price per gallon of the three olive oils shown. You must be sure the price shown is for the same unit. In this case, the unit is a gallon.

APPLICATIONS:

Indicate the best buy for each pair of items.

Note: Students need only compare the unit prices making sure that the units are the same. They do not have to do any computation or conversion.

1.

UNIT PRICE	YOU PAY
$2.49 per pt	$1.09 7 oz
BRAND A	

or

UNIT PRICE	YOU PAY
$2.39 per pt	$1.79 12 oz
BRAND B	

2.

UNIT PRICE	YOU PAY
$1.78 per lb	89¢ 8 oz
BRAND C	

or

UNIT PRICE	YOU PAY
$1.59 per lb	$1.49 15 oz
BRAND D	

3.

UNIT PRICE	YOU PAY
66¢ per lb	$1.98 3 lb
BRAND E	

or

UNIT PRICE	YOU PAY
69¢ per lb	69¢ 1 lb
BRAND F	

4.

UNIT PRICE	YOU PAY
$1.60 per qt	30¢ 6 oz
BRAND G	

or

UNIT PRICE	YOU PAY
$1.54 per qt	77¢ 1 pt
BRAND H	

5.

UNIT PRICE	YOU PAY
99.5¢ per qt	$1.99 $\frac{1}{2}$ gal
BRAND X	

or

UNIT PRICE	YOU PAY
$1.25 per qt	$1.25 1 qt
BRAND Y	

SUMMARY:

a. What is meant by the term "best buy"?

b. What is the easiest way to determine the best buy?

c. How many ounces are there in 1 pint? 1 quart? 1 gallon?

LESSON 3

AIM: To compute the actual price from the unit price

PERFORMANCE OBJECTIVES: Students will be able to:

- state the relationship between pounds and ounces in weight.
- find the product of two decimal numbers, and round off to the nearest hundredth.
- round off a monetary amount to the nearest cent.
- apply the rule: actual price = unit price times weight (or quantity).

VOCABULARY:

New Terms: pounds, ounces, rounding-off to nearest hundredth (cent)
Review: unit price, actual price

CHALLENGE PROBLEM:

Your job in the supermarket requires you to compute the actual cost from the unit price. Suppose the unit price of an article is 60¢ per quart; what would a pint cost?

UNIT PRICE	YOU PAY
60¢	?
per qt	1 pt

DEVELOPMENT:

Note: Teacher may find it helpful to have actual supermarket unit price labels.

1. The teacher should help the students see that the relationship between quantities is most important in computing cost. In the challenge problem, 2 pt = 1 qt or 1 pt = 1/2 qt. Therefore, the cost should be 1/2 of 60¢ = 30¢.

2. Elicit the relationship between the quantities' actual price, unit price, and number of units. Students may be led to discover this relationship by working with several simple computational problems.

   a) If candy costs 30¢ per bar, what is the cost of 2 bars? 5 bars?

   b) At $1.50 per pound for jelly beans, what is the cost of 5 pounds of jelly beans? 0.5 pounds?

   Have students write the relationship in words: actual price equals the unit price times the number of units.

3. Before doing application A it will be necessary to review equivalent measures. Place the following table on the board:

Measures of Weight

1 lb = 16 oz

4. Do application A .

5. Many items are not sold in whole units, and they may be purchased as 1.1 lb or .84 lb, etc. The most common items are meat, vegetables, fruits,etc.

   For example;

UNIT PRICE	YOU PAY
$1.89	?
per lb	1.41 lb
CHUCK STEAK	

This can be computed as

$$\begin{array}{r} 1.89 \\ \times\ 1.41 \\ \hline 1\ 89 \\ 75\ 6 \\ 1\ 89 \\ \hline 2.66\ 49 \end{array}$$

Rounded off to the nearest cent you pay $2.66. Note placement of decimal point in answer. Discuss rounding off to nearest cent (nearest hundredth).

6. Do applications B and C.

APPLICATIONS:

A. Compute the actual price for each item:

UNIT PRICE	YOU PAY
$1.60 per lb	? $1\frac{1}{2}$ lbs
COOKIES	

UNIT PRICE	YOU PAY
80¢ per lb	? 12 oz
HARD CANDY	

UNIT PRICE	YOU PAY
$2.00 per qt	? 24 oz
LEMON PIE	

UNIT PRICE	YOU PAY
$2.99 per lb	? 3 lb
CANNED HAM	

B. Compute the actual price to the nearest cent for each item:

UNIT PRICE	YOU PAY
$1.49 per lb	? 2.4 lb
SAUSAGE	

UNIT PRICE	YOU PAY
$2.99 per lb	? .89 lb
FLOUNDER	

UNIT PRICE	YOU PAY
65¢ per lb	? 3.24 lb
CHICKEN	

UNIT PRICE	YOU PAY
69¢ per lb	? 1.12 lb
APPLES	

C. Round off to the nearest cent.

1. $ 9.234	6. $ 0.893
2. $ 6.789	7. $ 1.874
3. $ 3.129	8. $10.895
4. $23.013	9. $36.553
5. $ 7.407	10. $123.445

SUMMARY:

A. To find the actual cost of an item given the unit price and weight, you:

1. change each to the same unit of measure.
2. multiply the unit price by the number of units in the actual weight.

B. Review converting units of weight.

LESSON 4

AIM: To compute the unit price from the actual price

PERFORMANCE OBJECTIVES: The student will be able to:

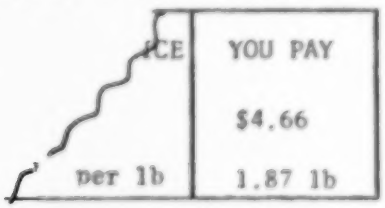
- find the quotient of a decimal number (monetary value) by a whole number expressed to the nearest cent.
- find the quotient of one decimal number by another to the nearest hundredth.
- apply the rule: unit price = actual price divided by the weight (or quantity).

VOCABULARY:

Review: unit price, actual price, weight, unit, round-off

CHALLENGE PROBLEM:

Chuck steak is on sale in the C Supermarket. The unit price label was torn off a package of meat and only the second half remains.



What was the sale price of the steak per pound?

DEVELOPMENT:

1. In yesterday's lesson, the actual price was obtained by multiplying the unit price by the weight. Elicit that today's challenge asks us to do the opposite. Recall that division is the opposite of multiplication.

$$
\begin{array}{r}
\$2.491 \\
1.87\overline{)4.66.000} \\
3\ 74 \\
\hline
92\ 0 \\
74\ 8 \\
\hline
17\ 20 \\
16\ 83 \\
\hline
370
\end{array}
$$

The sale price is $2.49 per pound.

Thus, the sale price of chuck steak was $2.49 per lb, and 1.87 lb would cost $4.66.

2. The teacher should provide enough applications of this type so that students can learn to divide decimal fractions.

APPLICATION:

Find the unit price for each item.

1.

UNIT PRICE	YOU PAY
?	$3.57
per lb	3 lb

2.

UNIT PRICE	YOU PAY
	65¢
per lb	8 oz

3.

UNIT PRICE	YOU PAY
	$4.32
per lb	1.73 lb

4.

UNIT PRICE	YOU PAY
	$2.36
per lb	4.12 lb

5.

UNIT PRICE	YOU PAY
	$1.35
per lb	1 lb

6.

UNIT PRICE	YOU PAY
	$1.98
per lb	2.25 lb

GENERAL MATHEMATICS CURRICULUM CORRELATION CHART

Fundamentals of Math - I

General High School Mathematics (New York State bulletin) *Content*	I. Formulas	II. Introduction to Algebra	III. Metric System	IV. Geometry	V. Money Management	VI. Statistics and Probability
I. INTEGERS						
A. Whole numbers						
1. Addition	1,2,3	1		3,10,11	7,11	
2. Subtraction	1,2,3	1		3	11	
3. Multiplication	1,2,3	1	√	5	11	
4. Division	1,2,3	1	√	14	5,6	
5. Order of operations	3,4					
6. Distributive property	4					
7. Commutative property		10				
B. Integers on the number line		6,7				
C. Addition on the number line		10				
D. Additive inverse		11				
E. Subtraction on the number line		6				
F. Order on the number line		2				
G. Rules for adding and subtracting signed numbers		11				
H. Mathematical phrases and sentences						
1. Concept and translation	2,6					
2. Truth or falsity		1				
3. Replacement and solution sets	2					
I. Problem solving						
1. Formulas and forming open sentences	7					
2. Evaluating formulas	7					
3. Finding solutions						
J. Multiplication of integers						
K. Division of integers						
L. Exponents, squares and square roots	5,6		2	5,6		
M. Miscellaneous						
1. Rounding off (including						

| | | |

IV. GEOMETRY												
A. Basic concepts												
1. Naming line segments, angles, and figures		4		2,3,7								
2. Angle measurement and use of protractor				2								
B. Constructions				9								
C. Metric system												
1. Estimation, which units to use, prefixes			1,8,9									
2. Basic units and conversion within system			3-10									
D. Perimeter (Polygon)												
1. Square				10,14								
2. Rectangle				10,14								
3. Triangle				10,14								
E. Area												
1. Square												
2. Rectangle												
3. Parallelogram												
4. Triangle												
5. Trapezoid												
F. Circle												
1. Definitions, points, and lines				9								
2. Circumference				12,13,14								
3. Area												
G. Pythagorean theorem				5,6,								
H. Triangles												
1. Definition and types, naming				1,4								
2. Sum of the angles				3								
I. Polygons												
1. Definition and types, naming				7,8								
V. RATIO, PROPORTION, AND PERCENT												
Scale drawing												
1. Ratio												
a. Meaning												
b. Reducing word problems												
2. Proportion												

a. Meaning														
b. Solving														
c. Word problems														
3. Percents														
a. Meaning														
b. Word problems														
c. Percent ↔ decimal														
d. Percent ↔ fraction														
VI. PROBABILITY AND STATISTICS														
1. Probability--Definition							6,7,11							
Sample space							9,10							
2. Statistics --Definition							1							
a. Organization of data							2							
b. Interpretation of data														
a. Mode							3							
b. Mean							4							
c. Median							5							
VII. CONSUMER AND JOB-RELATED MATHEMATICS														
1. Unit pricing					2-6									
2. Conversion of units					2-6									
3. Earning money					7,9,10									
4. Interest problems														
5. Discounts and sales tax														
6. Making change					1									
7. Banking vocabulary		1												
8. Installment buying					11									
9. Number of hours between two clock times					8									
10. Taximeter rates					12									
11. Tips														

* Numbers refer to lessons in respective units of Fundamentals of Mathematics - I.

7.

UNIT PRICE	YOU PAY
	$3.50
per lb	3 lb 8 oz

8.

UNIT PRICE	YOU PAY
	65¢
per gal	32 fl oz

9.

UNIT PRICE	YOU PAY
	$1.25
per oz	8.5 oz

10.

UNIT PRICE	YOU PAY
	$.47
per lb	4 oz

SUMMARY:

To find the unit price of an item, given the actual price and weight (or quantity):

(1) Express the weight in the proper units;
(2) Divide the weight into the actual price;
(3) Round off to the nearest cent.

LESSON 5

AIM: To compute the weight of an item, given the actual price and unit price

PERFORMANCE OBJECTIVES: The student will be able to:

- apply the rule: weight (or quantity) = actual price divided by unit price.

VOCABULARY:

Review: unit price, actual price, weight, quantity

CHALLENGE PROBLEM:

How many pounds did a sirloin steak weigh if the unit price is $2.49 per lb and the actual price of the steak was $3.82?

DEVELOPMENT:

1. This is the final set of examples possible with unit pricing. The challenge problem should be set aside and simpler problems done in order to establish the pattern for solving these.
2. Short exercises, like those below, should be done orally.

UNIT PRICE	YOU PAY
$1.50 per lb	$3.00 ? lb

UNIT PRICE	YOU PAY
69¢ per lb	$2.07 ? lb

UNIT PRICE	YOU PAY
26¢ per 4 oz	13¢ ? oz

From these simple examples, the students should see that they can divide the actual price by the unit price to determine the weight of the item. or weight = $\frac{\text{actual price}}{\text{unit price}}$

3. The students should generalize the result and solve the challenge problem.

4. Do applications.

APPLICATIONS:

A. Find the number of units of each item to the nearest hundredth of a unit:

UNIT PRICE	YOU PAY
45¢ per lb	90¢ ? lb

UNIT PRICE	YOU PAY
19¢ per oz	44¢ ? oz

UNIT PRICE	YOU PAY
$1.29 per lb	$3.87 ? lb

UNIT PRICE	YOU PAY
45¢ per lb	67¢ ? oz

UNIT PRICE	YOU PAY
69¢ per lb	$1.38 ? lb

UNIT PRICE	YOU PAY
29¢ per oz	94¢ ? lb

UNIT PRICE	YOU PAY
$1.394 per gal	$6.97 ? gal

UNIT PRICE	YOU PAY
$1.19 per lb	$5.01 ? lb

UNIT PRICE	YOU PAY
17¢ per oz	68¢ ? oz

UNIT PRICE	YOU PAY
$1.428 per gal	$10.00 ? gal

SUMMARY:

To find the weight (or quantity) of an item given the unit price and the actual price, divide the actual price by the unit price.

$$\text{weight or quantity} = \frac{\text{actual price}}{\text{unit price}}$$

LESSON 6

AIM: To review unit pricing

PERFORMANCE OBJECTIVES: The student will be able to:

- calculate a missing unit price, weight or quantity, or actual cost, given the other two.

VOCABULARY:

unit, price, weight, quantity, actual

CHALLENGE PROBLEM:

Find the missing quantity in each unit price ticket shown below. Show all work.

DEVELOPMENT:

1. The attached chart should be rexographed and distributed to the students. The students should do and review the challenge problem before completing the entire sheet of examples.

UNIT PRICE	YOU PAY
\$1.14 per $\frac{1}{2}$ gal	? 1 qt
MILK	

UNIT PRICE	YOU PAY
? per doz	45¢ $\frac{1}{2}$ doz
EGGS	

UNIT PRICE	YOU PAY
\$1.56 per lb	78¢ ? lb
BUTTER	

A. Fill in the missing quantity in each line of the chart.

ITEM	UNIT PRICE	WEIGHT or QUANTITY	ACTUAL COST
1. Coffee	$3.98 per lb	2 lb	
2. Peanut butter	$1.59 per lb	8 oz	
3. Canned beans	$ .78 per lb		39¢
4. Sugar	$ .79 per lb		$3.95
5. Cereal		12 oz	99¢
6. Cookies		.75 lb	$1.50
7. Flour	89¢ per 5 lb	2 lb	
8. Apples		1.3 lb	$1.29
9. Tea bags	99¢ per 100	125	
10. Franks	$1.39 per lb		$4.17
11. Ground round	$2.09 per lb	1.54 lb	
12. Sausage		12 oz	$1.09
13. Chicken	$ .79 per lb		$3.23
14. Sirloin		2.15 lb	$7.07

B. Which is the best buy?

1. Ice cream at 1 qt for 89¢,or 1 gal for $1.98?

2. Cooking oil at 1 pt for 99¢,or 24 oz for $1.50?

LESSON 7

AIM: To compute salary based on an hourly wage

PERFORMANCE OBJECTIVES: The student will be able to:

- define "gross wage."
- complete a payroll table which does not include any overtime pay.

VOCABULARY:

New Terms: gross wage, payroll

Review: product

CHALLENGE PROBLEM:

Jim Elliott was employed as a part-time waiter at A & R Fast Foods. He worked 2 hours, 3 hours, 2 hours, 4 hours, 5 hours and 8 hours on Monday through Saturday, respectively. If his hourly wage was $3.40, what was his total salary for the week?

DEVELOPMENT:

1. Using the challenge problem, elicit the fact that the product of the hourly wage and the number of hours worked is equal to the total salary. Define the total salary before deductions (city, state and federal taxes; social security; union dues, etc.) as the gross wage.
2. Solve the challenge problem with the class.
3. Do Application A.

APPLICATIONS:

A. Jim Elliott was one of seven employees at A&R Fast Foods. Below is a chart listing the payroll of the employees. Have the students complete the missing information. Elicit from students that $\frac{1}{2}$ hour = 0.5 hour, $\frac{1}{4}$ hour = 0.25 hour, etc.

CHART I

A&R FAST FOODS PAYROLL

NAME	M	T	W	T	F	S	TOTAL HOURS	RATE PER HR	GROSS WAGE
Elliott, James	2	3	2	4	5	8		$3.40	
Garcia, Maria	8	8	10	9	10	4		$3.80	
Haas, Eric	8	8	8	8	8	8		$4.00	
Rodriquez, Jose	5	5	5	7	7	$6\frac{1}{2}$		$3.50	
Ruben, Howie	5	7	6	6	8	$4\frac{3}{4}$		$3.70	
Slack, Mary	8	8	$8\frac{1}{2}$	7	8	$8\frac{1}{2}$		$3.60	
Wilson, Tom	9	9	$8\frac{1}{2}$	5	7	$8\frac{3}{4}$		$3.90	

TOTAL _____

SUMMARY:

Review how to determine the gross wage based on an hourly wage.

CLIFF-HANGER:

John arrived at work at 8:00 a.m., left for lunch at 11:30 a.m., returned from lunch at 12:30, and left for home at 4:30 p.m. How much money did John earn for the day if his hourly wage is $4.50?

LESSON 8

AIM: To read a time card

PERFORMANCE OBJECTIVES: The student will be able to:

- determine the number of hours between two given clock times, including a.m. and p.m. times.

VOCABULARY:

New Terms: time card, a.m., p.m.

CHALLENGE PROBLEM:

A part time worker arrived at her job at 10:15 a.m. and left at 2:45 p.m. How many hours did she work that day?

DEVELOPMENT:

1. Discuss challenge problem. Elicit a need for developing a technique for finding the difference between two clock times. Recall cliff-hanger from the previous lesson noting that computation was simplified because each pair of times was either a.m. or p.m. Discuss calculations.

11:30 a.m. means 11 hr 30 min after midnight

8:00 a.m. means $-\underline{8 \text{ hr } 0}$ min after midnight

3 hr 30 min time worked before lunch

likewise:

4:30 p.m. means 4 hrs 30 min after noon

12:30 p.m. means $-\underline{0 \text{ hrs } 30 \text{ min}}$ after noon

4 hrs 0 min is time worked after lunch

Total number of hours can now be computed

$$\begin{array}{r} 3 \text{ hr } 30 \text{ min} \\ +4 \text{ hr} \\ \hline 7 \text{ hr } 30 \text{ min} \end{array} = 7\frac{1}{2} \text{ hr}$$

and John's earnings for the day calculated as

$$7\frac{1}{2} \times 4.50 = \frac{15}{2} \times \frac{4.50}{1} = 33.75$$

2. Elicit that either noon or midnight may be used as the reference point for the cliff-hanger. Keeping that in mind, return to the challenge problem.

   12:00 noon means 12 hrs 0 min after midnight

   10:15 a.m. means 10 hrs 15 min after midnight

   Have students recognize necessity of exchanging 1 hr for 60 min so that 12 hrs 0 min = 11 hrs 60 min.

   Thus the problem becomes:

   11 hrs 60 min
   -10 hrs 15 min
   1 hr 45 min - hours worked before noon.

   Time worked after noon may be computed as follows:

   2:45 p.m. means 2 hrs 45 min after noon
   -12:00 noon means 0 hrs 0 min after noon
   2 hrs 45 min - hours worked after noon

   Total number of hours is found by adding hours worked before noon + hours worked after noon. Thus

   1 hr 45 min
   + 2 hr 45 min
   3 hr 90 min = 4 hr 30 min

3. Do application A.

4. Have the students consider the problem of a part-time worker who does not get paid for "break-time." Compute the actual number of hours worked by someone who arrives at 9:15 a.m., leaves at 2:30 p.m. and gets a 25-minute unpaid break.

   Solution: 9:15 to noon is 2 hr 45 min
   + noon to 2:30 is 2 hr 30 min
   Total time in is 4 hr 75 min
   - 25 min
   4 hr 50 min

   NOTE: Encourage students not to change 4 hrs 75 min to 5 hrs 15 min as they would have to undo this when they subtract.

All exchanges should be delayed and a final exchange made at the end of the problem, if necessary.

5. Do application B.

APPLICATIONS:

A. Find the number of hours and minutes worked between the times listed.

1. 8:30 a.m. to 11:45 a.m.
2. 1:15 p.m. to 5:00 p.m.
3. 7:45 a.m. to 8:30 a.m.
4. 8:45 a.m. to 5:15 p.m.
5. 7:15 p.m. to 12:00 midnight

B. Complete the time card for Derek Spears.

NAME: SPEARS, DEREK WEEK ENDING: JUNE 30

	Time In	Time Out	Hours Elapsed	Lunch Time	Hours Worked
Monday	9:00 a.m.	5:00 p.m.	8 hrs 0 min	45 min	_____
Tuesday	8:00 a.m.	5:00 p.m.	_____	1 hr	_____
Wednesday	8:30 a.m.	4:00 p.m.	_____	30 min	_____
Thursday	8:15 a.m.	4:30 p.m.	_____	50 min	_____
Friday	8:35 a.m.	5:15 p.m.	_____	1 hr	_____

Total hours worked _____

SUMMARY:

1. Review procedure for calculating difference between two clock hours.
2. Consider the following problem.
   A bus leaves Washington, D.C. at 10:40 a.m. and arrives in New York City at 4:25 p.m. What is the travel time?

LESSON 9

AIM: To compute salary based on hourly and overtime wages

PERFORMANCE OBJECTIVES: The student will be able to:

- find $1\frac{1}{2}$ times any dollar amount to the nearest cent.
- determine a worker's overtime hours.
- complete a payroll table which includes time and a half overtime pay.

VOCABULARY:

New Terms: overtime, total gross wages

Review: hourly wage

CHALLENGE PROBLEM:

In some businesses, overtime is paid to employees for work in excess of 35 hours. The standard overtime rate is $1\frac{1}{2}$ times the worker's regular hourly rate ("time and a half"). If Jim Elliott's hourly wage is \$3.40, determine his hourly overtime wage.

DEVELOPMENT:

1. Using the challenge problem, lead the students to the generalization that $1\frac{1}{2}$ x hourly wage = overtime hourly wage.

$$1\frac{1}{2} \times 3.40$$
$$1.70$$
$$\frac{3}{1} \times \frac{3.40}{1} = \$5.10$$

2. Do application A.
3. Have the students determine the steps involved in completing the payroll chart listed below. These steps are:
   i. Calculate the total number of hours worked.
   ii. Determine the number of overtime hours.
   iii. Compute regular wage by multiplying the regular rate per hour by the number of regular hours.
   iv. Calculate overtime rate per hour.

v. Compute overtime wage by multiplying overtime rate per hour by the number of overtime hours.

vi. Add the regular wage and overtime wage to find the total wage.

4. Do application B.

B. Consider 35 hours as a regular work week.

A.

A & R FAST FOODS PAYROLL

EMPLOYEE	M	T	W	T	F	S	TOTAL HRS.	REG. HRS.	REG. RATE PER HR.	REG. WAGE	OVER-TIME HRS.	OVER-TIME RATE PER HR.	OVER TIME WAGE	TOTAL GROSS WAGES PER HR.
ELLIOTT, JAMES	2	3	2	4	5	8	24	24	$3.40	$81.60	0	$5.10	0	$81.60
GARCIA,. MARCIA	8	8	10	9	10	4	49	35	$3.80		14			
HAAS, ERIC	6	8	8	8	8	8	48		$4.00					
RODRIQUEZ,JOSE	5	5	5	7	7	6	35		$3.50					
RUBIN, HOWIE	5	7	6	6	8	4	36		$3.70					
SLACK, MARY	8	8	8	7	8	8	47		$3.60					
WILSON, TOM	9	9	8	5	7	8	46		$3.90					

TOTAL PAYROLL _____

B.

TIME CARD

NAME: CHESTER TATE JOB TITLE: CLERK

HOURLY RATE: $4.18 WEEK ENDING: DEC. 5

	IN	OUT	LUNCH	IN	OUT	DAILY HOURS
MONDAY	7:00 a.m.	12:00 noon		1:15 p.m.	5:00 p.m.	$8\frac{3}{4} = 8.75$
TUESDAY	8:15 a.m.	12:00 noon		1:00 p.m.	5:30 p.m.	
WEDNESDAY	7:00 a.m.	11:15 a.m.		1:00 p.m.	6:45 p.m.	
THURSDAY	6:45 a.m.	12:00 noon		1:30 p.m.	5:15 p.m.	
FRIDAY	8:00 a.m.	12:00 noon		1:00 p.m.	3:00 p.m.	

TOTAL WEEKLY HOURS _____

Chester Tate's time card indicates the number of hours he worked for the week ending Dec. 5. Complete the card to find his total weekly hours. Hours over 37.5 are paid time and a half. Find Chester's wage for that week.

APPLICATIONS:

Have the students determine the overtime hourly wage for each employee in Lesson 7, Chart I.

SUMMARY:

Review how to determine regular and overtime hours, regular wage, overtime wage and total gross wages.

LESSON 10

AIM: To compute salary based on piecework or a piece-rate plan

PERFORMANCE OBJECTIVES: The student will be able to:

- name several jobs which pay by the "piece-rate plan."
- complete a payroll table where the workers are paid by the piece.

VOCABULARY:

New Terms: piecework (piece-rate plan), subscriber

Review: payroll, wage

CHALLENGE PROBLEM:

Mary is a sewing machine operator who earns her pay by piecework. If she earns \$0.25 to sew in a sleeve, \$0.15 to sew in a collar, how much would she earn if she sewed in 46 sleeves and 32 collars?

DEVELOPMENT:

1. The challenge problem should lead to a discussion of the meaning of piecework, its value and shortcomings. The problem should be solved as follows:

\$ 0.25	\$ 0.15	and	\$11.50	
X 46	X 32		+ 4.80	
\$ 11.50	\$ 4.80		\$16.30	Mary Earns \$16.30

2. Do application A.

3. The students can discuss other types of work which involve piece-rates. A suggested exercise follows:

   Frank Anders is a newspaper delivery boy for the local paper. He receives 15¢ a week for each subscriber. If Frank's route consists of 45 subscribers, how much does the newspaper pay him per week?

   After this exercise is solved, the teacher can expand the problem by increasing the number of subscriptions and possibly adding tips for good service to Frank's salary.

For example:

a. Suppose 15 more subscribers join Frank's route. What is his new salary?

b. If he receives tips of 25¢ from 30 subscribers, 35¢ from 10 others and 15¢ from the remaining 5, what are his earnings in tips?

4. The students can discuss other types of work which involve piece-rates.

5. Do application B.

APPLICATIONS:

A. 1. Marie Tashji works in a factory as a machine operator. She receives $.35 for every inserted zipper. If she sews 190 zippers in one day, how much does she earn?

2. Rosita Hernandez also works in a factory as a machine operator. She receives $0.50 for every set of sleeves sewed on a robe. If she completes 196 sets of sleeves in one day, how much does she earn by the end of the day?

B. Compute the missing information for the following payroll. Will the company have enough money to pay its employees, if it has $760 in its payroll account?

WEEKLY PAYROLL

EMPLOYEE	WAGE PER PIECE	TOTAL # OF PIECES	TOTAL WAGES
BAKER, MARY	.75	200	
CODD, JANE	1.50	50	
HERNANDEZ, MARIA	3.00	50	
MORRIS, SHERI	2.25	50	
TURNER, BETTY	.50	150	
WONG,LISA	3.00	40	
WILLIAMS, KAREN	.60	100	
		TOTAL	

SUMMARY:

Review the concept of piece-rate and how to compute salary based on a piece-rate plan.

LESSON 11

AIM: To find the cost of an item purchased on the installment plan

PERFORMANCE OBJECTIVES: The student will be able to:

- explain why people purchase items on the installment plan.
- compute the total cost of an item purchased on the installment plan.
- compute the finance charge on an installment purchase.

VOCABULARY:

New Terms: installment, installment plan, cash price, total cost, down payment, finance charge, service charge, revolving credit charge, carrying charge

CHALLENGE PROBLEM:

Darryl wishes to purchase a new stereo system. He saw one at the store for $600, but has only $100 now. The store manager tells him that he can buy the stereo for $100 down and 12 monthly payments of $50. What will the total cost of the stereo be if it is purchased in this manner?

DEVELOPMENT:

1. Elicit that this type of purchasing is called buying on time or installment buying. Why is this method of purchasing advantageous to many people? What are some items that are commonly purchased on the installment plan?
2. Discuss the challenge problem and define cash price, down payment, installments or (monthly) payments, amount financed, finance charge and total cost.
   Elicit that total cost = down payment + number of installments times amount per installment.
   Elicit that total cost - cash price = finance charge (service charge, revolving credit charge, carrying charge).
3. The teacher should provide the students with a variety of

applications in the form of verbal problems. Students should be able to compute any quantity (total cost, down payment, number of payments, monthly payment or finance charge) given any three of these.

APPLICATIONS:

A. Complete the following table to find the total cost of each item.

Item	Down Payment	Number of Installments	Amount of Installment	Total Cost
T.V. set	$55	18	$30	
Air conditioner	$95	24	$16.20	
Refrigerator	$150	36	$16.85	
Sewing machine	$28	30	$10.76	

B. Mrs. Bradford bought a radio marked $125 on the installment plan. She paid $25 down and made 6 installment payments of $18.95 each. What was the total cost of the radio. How much was the finance charge?

C. The total cost of a videorecorder purchased on the installment plan was $970. Mr. McCarthy paid $127 down and paid the remainder in 12 equal monthly installments. How much was each installment?

SUMMARY:

Review vocabulary of installment buying.

CLIFF-HANGER

Teresa bought an item for $20 down. Her payments for the first six months were $20.25 per month, and for the next six months were $15.75 per month. What was the total amount paid for the item?

LESSON 12

AIM: To compute payments based on a step function

PERFORMANCE OBJECTIVES: The student will be able to:

- provide real-life examples of "step function" price scales.
- determine the number of units at each price.
- compute the total price by adding the initial charge to the product of the unit rate and the number of units at that price.

VOCABULARY:

New Terms: step function, initial charge, additional charge, rate schedule

CHALLENGE PROBLEM:

The first-class rates for mailing a package at the post office are as follows:

$ .18 for the first ounce

$ .17 for each additional ounce

What is the cost for mailing a package weighing 3 ounces?

DEVELOPMENT:

1. Discuss the challenge problem and elicit other situations in which rates are changed when additional units are added. Some examples might include taxi fares, telephone rates, postal rates, electric rates, etc.
2. Solve challenge problem: $ .18 + 2 ($ .17) = $ .18 + $ .34 = $ .52 Elicit the necessity for first determining the number of ounces at each rate. Compute the total charge by adding the initial charge to the additional charge.
3. Do application A.
4. Return to cliff-hanger of Lesson 11. Note that this problem involves one initial charge plus two additional charges. Discuss and solve this problem: $20 + 6 ($20.25) + 6 ($15.75) = $236.00

5. Do application B.

APPLICATIONS:

A. 1. The cost of a long-distance telephone call is as follows:

$1.02 for the first two minutes

.45 for each additional minute

What is the total cost for a six minute long distance call?

2. In a certain city the taxi rates are $1.25 for the first one fourth ($\frac{1}{4}$) mile and $.75 for each additional half ($\frac{1}{2}$) mile, or part thereof. Find the cost of a $1\frac{1}{4}$ mile trip. (Teacher may discuss cost of a $1\frac{1}{2}$ mile trip.)

B. The Hickmon family pays for its electricity according to the rate schedule given. The rate schedule indicates the cost per kilowatt-hour. It is used for each monthly bill. What is the cost of the first 30 kilowatt hours? The next 170? The next 600?

RATE SCHEDULE

Kilowatt-Hours	Cost/Kilowatt-hour
First 30	\$0.095 ($9\frac{1}{2}$¢)
Next 170	0.035
Next 600	0.03
Next 700	0.026
Over 1500	0.015

If the Hickmon family used 423 kilowatt-hours for one month, what is the amount of the bill (disregard taxes and fuel adjustments)?

Solution:

423
- 30 @ $ .095
393
-170 @ $ .035
223 @ $ .03

30		170		2.23		2.85
X .095	+	X .035	+	X .03	=	5.95
$2.850		$5.950		$6.69		6.69
						$15.49

SUMMARY:

Review process of computing payments based on a step function.

UNIT V : MONEY MANAGEMENT

SUGGESTED TEST ITEMS

*1. John spent $6.43 in a store. What was his change from a $20 bill?

2. If 5 gallons of gasoline cost $6.80, what is the cost per gallon?

3. Vicki bought 8 ounces of chocolate at $3.20 per pound. How much should she pay?

4. Mary purchased 1.1 pounds of apples at 69¢ a pound. Find the cost of this purchase to the nearest cent.

*5. If wallpaper costs $8.95 a roll, what is the cost of 9 rolls?

6. Which is the better buy, brand A or brand B?

UNIT PRICE	YOU PAY
59¢	30¢
per 1 lb	$\frac{1}{2}$ lb
BRAND A	

UNIT PRICE	YOU PAY
56¢	14¢
per 1 lb	4 oz
BRAND B	

7. Fill in the missing quantity on each unit price label.

UNIT PRICE	YOU PAY
$1.88	?
per 1 lb	4 oz
BUTTER	

UNIT PRICE	YOU PAY
$1.20	89¢
per 1 lb	?
ROLLS	

UNIT PRICE	YOU PAY
?	63¢
per gal	1 qt
LEMONADE	

*8. A bus leaves its terminal at 10:30 a.m. and arrives at its destination at 1:15 p.m. on the same day. How long does the trip take?

*9. Jamie earns $4.00 per hour at his part-time job. If he works $6\frac{1}{2}$ hours one day, how much does he earn?

10. Sam Elliott worked 39 hours in one week. His hourly rate of pay is $5.40. He receives time and a half for work over 35 hours. Compute his salary for that week.

11. Stephanie delivers circulars for a local supermarket. She receives $.07 for each address on her route. If there are 275 addresses to which she delivers, how much does she earn?

*12. Cathy bought a mini-bike with $50 down payment and 10 monthly installments of $25 each. What was the total cost of the mini-bike?

*13. The cost of duplicating a page is as follows:

$13.00 for the first 100 copies

.10 for each additional copy

How much would it cost to have 150 copies made?

*14. Multiply: 3.6

$\times$ 5.4

*15 Divide: 8) 40.96

16. Subtract: 4001

$-$ 375

Unit VI. Probability and Statistics

LESSON 1

AIM: To introduce statistics as the science of collecting, organizing and interpreting data

PERFORMANCE OBJECTIVES: The student will be able to:

- provide examples in which analysis of data is needed to make decisions in many fields: politics, retailing, science.
- define "statistics" as the science of collecting, organizing, and interpreting data.

VOCABULARY:

New Terms: survey, statistics, statistician, data

CHALLENGE PROBLEM:

Mr. Jackson is the manager of a shoe store. He knows that a particular shoe style will be in demand for the upcoming season. How can he order shoes so that he has an adequate supply of popular shoe sizes?

DEVELOPMENT:

1. From the challenge problem elicit the fact that Mr. Jackson can use previously collected data (old orders and inventories) to determine his new order. Emphasize that statistics is the science of collecting, organizing and interpreting data. It plays an essential role in economics, science, medicine, politics, insurance, etc. Have each member of the class complete the Class Survey Data Form. Inform them that they will be using the information from their own responses on the data form. The next few lessons will involve them in the role of statistician: collecting, organizing and interpreting data.

CLASS SURVEY DATA FORM

Directions: Answer every question.

1. Favorite ice cream flavor
2. Shoe size
3. Height (in total inches)
4. Birth date (month, day, year)
5. Favorite indoor sport
6. Favorite outdoors sport
7. Favorite male singer
8. Favorite female singer
9. Favorite color
10. Jeans size (girl)
11. Jeans size (boy) answer one which applies
12. Social security number
13. Favorite television show
14. Favorite season
15. Number of children in your family, including yourself
16. Favorite television personality
17. Favorite car (American or foreign)
18. Favorite subject taken in high school
19. Number of years you plan attending (0, 1, 2, 3, 4, 5, 6, more than 6) school after being graduated from high school
20. Favorite brand of sneakers
21. Favorite musical group
22. Favorite disc jockey
23. Method(s) of travel you use to (walk, bus, car, bike, motorcycle) go to school
24. Favorite candy bar
25. Weight in pounds

SUMMARY:

Review the purposes of statistics and the job of a statistician.

LESSON 2

AIM: To organize and interpret statistical data using a frequency table

PERFORMANCE OBJECTIVES: The student will be able to:

- read a set of random data, and set up a system of classes (categories) for the purpose of tallying.
- tally the data according to the scheme determined above.
- define "frequency."
- organize the data by completing a frequency table.

VOCABULARY:

New Terms: random, ordered, tally, frequency, frequency table

Review: data

CHALLENGE PROBLEM:

Refer back to the problem posed in the previous lesson in which Mr. Jackson had to order shoes. Present the data listed below:

DEVELOPMENT:

1. Mr. Jackson found the following data: Have the pupils discuss the pros and cons of presenting this information in this form.

MEN'S SHOE SIZES:

$8, 9, 9, 8, 9\frac{1}{2}, 9, 8, 9\frac{1}{2}, 10, 9, 8, 10, 10\frac{1}{2}, 9, 10, 10, 9, 10\frac{1}{2}, 11\frac{1}{2}, 9, 9\frac{1}{2}, 9\frac{1}{2}, 10\frac{1}{2}, 11\frac{1}{2}$

$10, 10\frac{1}{2}, 10, 9\frac{1}{2}, 10\frac{1}{2}, 10, 9\frac{1}{2}, 11\frac{1}{2}, 12, 8\frac{1}{2}, 9\frac{1}{2}, 11, 11, 9\frac{1}{2}, 11\frac{1}{2}, 11\frac{1}{2}, 12, 8\frac{1}{2}, 10, 8\frac{1}{2}$

$10, 11, 10, 10, 11, 10, 10\frac{1}{2}, 10\frac{1}{2}, 11, 10\frac{1}{2}, 11, 11, 11, 11, 11\frac{1}{2}, 10\frac{1}{2}, 12, 8\frac{1}{2}, 11\frac{1}{2}, 11\frac{1}{2}$

$12\frac{1}{2}, 10\frac{1}{2}, 12, 8\frac{1}{2}, 10\frac{1}{2}, 11, 11, 11\frac{1}{2}, 11\frac{1}{2}, 12\frac{1}{2}, 10\frac{1}{2}, 10\frac{1}{2}, 11, 11, 8\frac{1}{2}, 10\frac{1}{2}, 11\frac{1}{2}, 11\frac{1}{2}, 11\frac{1}{2}$

WOMEN'S SHOE SIZES

$5,6,6,7,9,8\frac{1}{2},7,7,7\frac{1}{2},6\frac{1}{2},9,9\frac{1}{2},10,8,8,7,7,7\frac{1}{2},6,5\frac{1}{2},4,4,4\frac{1}{2},8\frac{1}{2},7\frac{1}{2},9,9,$

$7\frac{1}{2},8,7,7,7,7,10,7,7\frac{1}{2},6,6,7,9,8\frac{1}{2},9,9,9,10,7\frac{1}{2},7\frac{1}{2},6,6,,4\frac{1}{2},4,7,7,7,9,9\frac{1}{2},$

$10,7,7,7\frac{1}{2},8,7\frac{1}{2},9,10,7\frac{1}{2},7\frac{1}{2}$

2. The pupils will probably suggest an arrangement which would be more helpful to Mr. Jackson such as a separate grouping by sizes for men and women. An ordered data sample could be set up in the following way: (define "tally" and "frequency.")

MEN-SHOE SIZE	TALLY	FREQUENCY
8	1111	4
$8\frac{1}{2}$	~~1111~~ 1	6
9	~~1111~~ 11	7
$9\frac{1}{2}$	~~1111~~ 111	8
10	~~1111~~ ~~1111~~ 11	12
$10\frac{1}{2}$	~~1111~~ ~~1111~~ ~~1111~~	15
11	~~1111~~ ~~1111~~ 111	13
$11\frac{1}{2}$	~~1111~~ ~~1111~~ 11	12
12	1111	4
$12\frac{1}{2}$	11	2

The teacher should lead the students to the conclusion that an ordered presentation of the data is much more useful than a random one. The table showing the ordered presentation is called a Frequency Table.

3. Using the example, have the students write in their notebooks the essential characteristics of the Frequency Table. These are a title and columns for data (ordered if appropriate).

Note: Some types of data cannot be ordered, for example, flavors of ice cream, colors, seasons, etc. Other types of data can be ordered, for example, shoe size, height, age, etc.

4. Additional examples may be selected from the Class Survey Data Form (questions 2, 3, 10, 11, 15, 25).

SUMMARY:

Review the need for and the essential parts of a frequency table.

LESSON 3

AIM: To define and use the concept of mode

PERFORMANCE OBJECTIVES: The student will be able to:

- define mode
- identify the mode from frequency tables, including the class survey data table.
- define bimodality

VOCABULARY:

New Terms: mode, bimodality

Review: frequency, data

CHALLENGE PROBLEM:

The following record represents the number of half-gallons of ice cream sold at John's Deli. Which ice cream flavor was the most popular?

John's Deli Ice Cream Sales

Flavor	Number Sold
Strawberry	30
Pistachio	15
Lemon/Lime Sherbet	5
Orange Sherbet	6
Chocolate	40
Vanilla	25
Chocolate Chip	37

DEVELOPMENT:

1. The most popular flavor of half-gallons sold is called the "mode." The students should see that no computation or ordering is involved in arriving at the mode (chocolate). They must simply find the item which appears the greatest number of times.

Note: The "mode" represents the most frequently-appearing item, not the frequency of this item. The "mode" is "chocolate," not "40."

Define: The mode is the item with the greatest frequency.

2. The data from the Class Survey Data Form (questions 1, 5-7, 13-18, 30, 23, 24) may be used to find the mode.

3. Do applications.

APPLICATIONS:

A. Using the tables shown, determine the mode of each distribution.

Table 1

Alton Factory's Salary Scale

Salary	# Employees
$ 8,000	12
$ 9,000	80
$ 10,000	15
$ 11,000	10
$ 15,000	5
$ 20,000	3
$100,000	1

Table 2

Regents Test Results for Mrs. Jones' Algebra Class

0-54	3
55-64	10
65-74	10
75-84	8
85-94	4
95-100	2

Note to Teacher: Table 2 demonstrates bimodality. This concept should be explained to the pupils. (Review use of prefix bi.) Not every distribution has a mode. There may be several items of equally high frequency.

SUMMARY: Review the concept and use of "mode" in statistics.

LESSON 4

AIM: To define and use the concept of mean

PERFORMANCE OBJECTIVES: The student will be able to:

- define mean as the sum of the values divided by the number of values (average).
- apply the definition to find the mean, given a set of data.

VOCABULARY:

New Terms: mean, average

Review: data

CHALLENGE PROBLEM:

John's math scores were 100, 50, 50 and 60. His father asked him how he was doing in the subject. John answered, "I'm passing."

1. How did John arrive at this conclusion?
2. What statistical tool of measurement did he use to arrive at his conclusion?
3. Did his response give an accurate picture of how he was doing in math? Why? (Note: Discuss the answer to this part of the challenge in application D.)

DEVELOPMENT:

Using the challenge problem, explain that the arithmetic average is called the "mean." Elicit from the class that the mean or arithmetic average is found by dividing the sum of the set of scores by the number of scores. Mean = $\frac{100 + 50 + 50 + 60}{4} = \frac{260}{4} = 65$

APPLICATIONS:

A. Have the students find the mean for each of the following sets of data:

1. 20, 30, 30, 70, 70, 70, 80
2. 60, 60, 70, 80, 70, 60, 100, 100

B. The teacher may select additional exercises from the Class Survey Data Form (questions 3, 25) and use this data to compute the mean.

C. In a March of Dimes fund drive, a freshman class collected the following contributions. Find the average (mean) contribution.

20¢, 20¢, 40¢, 25¢, 20¢, 25¢,
10¢, 5¢, 10¢, 10¢, 20¢, 15¢,
10¢, 10¢, 15¢, 20¢, 50¢, 30¢.

D. Use the same data in question C, but add the teacher's contribution of $5.00. What effect does this additional information have on the mean? (Use the discussion of this question as a method of responding to the challenge problem,part 3.)

E. Find the mean value for each of the following:

1. 400, 200, 300, 600
2. 20, 30, 60, 30, 20, 40
3. 15, 25, 35, 45, 35, 35, 70
4. $10,000, $8,000, $8,000, $7,000, $25,000

F. Does the mean of a set of data always equal the mode of the set? Write a set of data in which the mean and mode are equal and a set of data in which they are not.

SUMMARY:

Review the concept of the mean as an arithmetic average.

LESSON 5

AIM: To define and use the concept of median

PERFORMANCE OBJECTIVES: The student will be able to:

- arrange a set of data in numerical order
- define median
- apply the definition to find the median in given sets of data.

VOCABULARY:

New Terms: median, central tendency, halfway

CHALLENGE PROBLEM:

Using the following data, determine the average (mean) height of a team of PSAL baseball players: 60 inches, 70 inches, 72 inches, 60 inches, 62 inches, 60 inches, 62 inches, 70 inches, 60 inches.

DEVELOPMENT:

1. After finding the average height (64 inches), discuss the fact that the answer is not representative of any player's height; more than half of the players' heights are less than 64 inches. Elicit the fact that the three extreme values of 70 inches, 70 inches, 72 inches affected the mean, resulting in a distorted conclusion.

2. Develop the concept of "median." Have the students order the given data numerically as follows: 60, 60, 60, 60, 62, 62, 70, 70, 72. Pose the following questions:
   a. Which height is in the middle?
   b. How many heights are before it (smaller)? After it (larger)? Define this middle value as the "median." Here the median of 62 is a more realistic measure of central tendency.
   c. Have the students find the mode for this set of data. Discuss the differences among the three measures of central tendency.

3. Do application A.

4. Have the students construct the following frequency table. Data is sometimes presented in this form.

HEIGHT (INCHES)	FREQUENCY
60	4
62	2
70	2
72	1

From the frequency table, how can the median value of 62 be determined? Develop the fact that the total frequency is 9, and that the 5th value from either direction is in the middle and is therefore the median. Thus: 62 is the median score.

5. Do application B.

6. Ask students to find the median for the following heights: 60 inches, 64 inches, 70 inches, 60 inches. In this example there is an even number of items in the data set. Have students arrange the data in order: 60, 60 ↑ 64, 70. Indicate, using an arrow, that the median is halfway between the two middle values; therefore, it is the average mean of the two middle values, or 62.

7. Do applications C and D.

APPLICATIONS:

A. Find the median.

1. 72, 83, 85, 92, 88
2. 11, 9, 17, 13, 27, 29, 28
3. 19, 17, 19, 17, 19, 20, 20, 17, 18

B. Find the median.

Quiz Scores	Frequency
6	8
7	7
8	10
9	3
10	1

C. Find the median.

1. 400, 200, 600, 300, 600, 400
2. 20, 40, 80, 10, 50, 60, 30, 30
3. 15, 25, 10, 5, 30, 65, 10, 10

D. Find the median

Class sizes	Frequency
30	2
31	7
32	9
33	14
34	8
35	3
36	1

SUMMARY:

The following are weights in pounds of nine football players: 170, 210, 240, 210, 250, 230, 200,230,210.

a. Arrange the data into a frequency table.

b. Find the mode, mean, and median.

c. Discuss the significance of these three different measures of central tendency.

LESSON 6

AIM: To introduce probability as the science of chance

PERFORMANCE OBJECTIVES: The student will be able to:

- cite uses of "chance" or probability.
- guess (or predict) the outcomes of experiments with a spinner, dice, or coins.
- perform an experiment, tally the outcomes, and compare the outcomes to the predictions.

VOCABULARY:

New Terms: probability, chance, prediction, outcome, expected outcome, experimental data, trials

DEVELOPMENT:

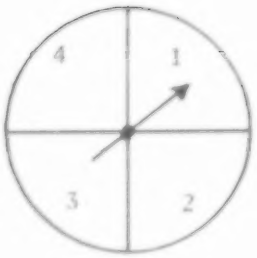
1. The teacher should provide some historical background to motivate the topic. For example, the science of probability began in gambling halls in the 1500's. Games of chance stirred the imagination of gamblers. The Italian physician and mathematician, Cardan, wrote the first book on probability called The Book on Games of Chance.

2. The purpose of this unit is to have students do simple experiments dealing with coins, dice spinners and any other objects, so that they may get the feeling that probability is just another way of using numbers to predict future outcomes based on past experiences.

EXPERIMENT I

The teacher should create a spinner as shown. Two players each take a turn at the spinner. The numeral at which the spinner stops tells the player his score for that turn. The scores for each player should be recorded in his own table (Table I). Eight spins should be recorded for each player.

TABLE I (EXPERIMENT I)



PLAYER

	TRIALS							
	1	2	3	4	5	6	7	8
1								
2								

The following questions can be posed to the class:

1. If the spinner stopped on number three, what score did the player get for the spin?
2. What is the greatest score that a player can get for one spin?
3. What is the least score that a player can get for one spin?
4. What numeral do you think would be most likely to occur? Least likely to occur? Why? (They should see that each number is equally likely.)
5. Is it just as hard to spin a one as a two? Why?
6. After the experiment of 8 spins has been completed, tabulate the experimental data. Compare this tabulation with the expected outcome. The expectation is two ones, two twos, two threes, two fours.
7. Continue the experiment with additional spins in sets of 4. After each set of spins (12, 16, 20, etc.), compare the experimental data with the expected outcome.
8. Should the experimental results for both players closely approximate each other? Why?

EXPERIMENT II:

Maria and Rena were deciding who would stay home and baby sit for their brother, Miguel. They decided to flip a coin ten times. The one with the greater number of heads would have to babysit. Did each have an equal chance not to babysit? Try the experiment to find out. Tabulate your results using the chart below.

TABLE II (EXPERIMENT II)

	TRIALS									
OUTCOMES	1	2	3	4	5	6	7	8	9	10
HEADS										
TAILS										

Would the experimental data have been different if the coin had been tossed 20 times? 30 times? This problem can be tested and recorded in class or at home , as time permits.

SUMMARY:

Review the concept that, through experiments, mathematicians can arrive at a fairly good estimate for predictions. Elicit similar types of experiments that students can perform on their own. This discussion could be one way to stimulate their curiosity and provide ground work for further study in probability.

LESSON 7

AIM: To define probability of success and failure as ratios

PERFORMANCE OBJECTIVES: The student will be able to:

- express a probability as the ratio of two quantities:

$$\frac{\text{number of successful outcomes}}{\text{total number of possible outcomes}}$$

- state the relationship between the number of outcomes, the number of successes, and the number of failures.
- use the rule that the probability of success plus the probability of failure equals 1. (Calculate the probability of an event given the probability of its complement.)
- reduce proper fractions to lowest terms.

VOCABULARY:

New Terms: ratio, comparison, numerator, denominator, equivalent fractional forms, success, failure

CHALLENGE PROBLEM:

If all the Delaney cards of the students present today are shuffled and placed in a pile, what is the probability that your card is on the top of the pile?

DEVELOPMENT:

1. Discuss the challenge problem. Elicit the fact that the probability of a successful event can be represented as a comparison or ratio of two numbers:

$$\text{probability (success)} = \frac{\text{number of successes}}{\text{number of possible outcomes}}$$

In similar fashion:

$$\text{probability (failure)} = \frac{\text{number of failures}}{\text{number of possible outcomes}}$$

Pose the question: What is the probability that your card will <u>not</u> be the top card?

2. Introduce symbolic notation and rewrite the definitions:

$P(S) = \frac{s}{s + f}$ $\qquad$ $P(F) = \frac{f}{s + f}$

s = number of successes $\qquad$ f = number of failures

$s + f$ = number of possible outcomes

Note: P (S) is read: the probability of success. Emphasize that the number of possible outcomes is the sum of the number of successes (desired outcomes) and the number of failures (all other outcomes).

3. Do Experiment I

4. Elicit from Experiment I that P (S) + P (F) = 1. Furthermore if P (S) is known, then P (F) can be calculated by subtracting P (S) from 1.

   For example: If John makes 7 out of every 10 "free throws," the probability that he misses a "free throw" is $1 - \frac{7}{10}$ or $\frac{3}{10}$.

   Review changing 1 to any equivalent fraction (in this case $\frac{10}{10}$).

5. Do Experiments II and III. It may be useful to use large model playing cards.

6. Refer to fractions such as $\frac{2}{10}$, $\frac{5}{10}$, $\frac{13}{52}$. Have students recognize that these fractions are generally expressed in lowest terms (i.e., $\frac{2}{10} = \frac{1}{5}$, $\frac{5}{10} = \frac{1}{2}$, $\frac{13}{52} = \frac{1}{4}$). Develop a procedure for reducing these ratios to the lowest terms (Note: The lesson which follows this lesson is devoted to reducing fractions in general.)

APPLICATIONS:

EXPERIMENT I

Complete the given chart

EXPERIMENT	OUTCOMES	SUCCESSFUL OUTCOMES	FAILURES	P(S)	P(F)	P(S)+P(F)
1. Toss a nickel	H, T	H	T			
2. Toss two coins	HH, HT, TT, TH	HH				
3. Spin the pointer						
4. Toss a die with six faces (cube).						
5. Toss a die with four faces (tetra hedron)	1, 2, 3, 4	even number				
6. Toss a die with eight faces (octahedron).	1, 2, 3, 4, 5, 6, 7, 8	1, 2, 3, 4	5, 6, 7, 8			
7. Pick a day at random	M, T, W, Th, F, Sa, Su	Monday	T, W, Th, F, Sa, Su			
8. Pick a day at random	M, T, W, Th, F, Sa, Su	Begin with "S"				
9. Pick a month at random		Has 30 days				
10. Pick a month at random		Has 31 days				

EXPERIMENT II

Complete the table assuming that one card can be drawn from ten cards consisting of a king of hearts and diamonds, jack of hearts; ace of spades, diamonds, hearts and clubs; two of hearts; four of spades, and six of hearts.

DRAW CONSIDERED A SUCCESS	P (S)	P (F)	P (S) + P (F)
King of hearts			
King of diamonds			
Red king			
Black king			
Ace of diamonds			
Red diamond			
Red card			
Two of spades			
Two of diamonds			
Black card			
Queen of hearts			
Four of spades			

EXPERIMENT III

Complete the table, asuming that a standard playing deck of 52 cards is used.

DRAW CONSIDERED SUCCESS	P (S)	P (F)	P (S) + P (F)
An ace			
A red card			
A black card			
A non-picture card			
A club			
A heart			
A red			
A black ace			
A picture card			
A one-eyed jack			

Note: It may be necessary to define "picture card," "one-eyed jack," etc.

LESSON 8

AIM: To reduce fractions to lowest terms

PERFORMANCE OBJECTIVES: The student will be able to:

- write a composite number as a product of two or more factors
- categorize numbers as prime or composite
- identify the GCF (greatest common factor) of a pair of numbers
- reduce a fraction to its lowest terms by dividing the numerator and denominator by their GCF.

VOCABULARY:

New Terms: common factor, GCF (greatest common factor), prime, composite, lowest terms

CHALLENGE PROBLEM:

One student said that the probability of choosing a Tuesday in February was $\frac{1}{7}$. Another student said that the answer should be $\frac{4}{28}$. A third student said that both answers were correct. A fourth student disagreed with everyone. How do you explain this?

DEVELOPMENT:

1. Discuss challenge problem with respect to years where February has more than 28 days. Elicit when February contains 28 days, the probability of $\frac{1}{7}$ is true. Show that $\frac{1}{7}$ and $\frac{4}{28}$ are equivalent fractions. Why? Students have previously raised fractions to equivalent forms in higher terms. Using this same procedure: $\frac{1}{7} = \frac{1 \times 4}{7 \times 4} = \frac{4}{28}$ and conversely $\frac{4}{28} = \frac{4 \times 1}{4 \times 7} = \frac{4}{4} \times \frac{1}{7} = \frac{1}{7}$, then elicit that both the numerator and denominator were expressed as a product of two factors, and that 4 was a common factor of both. (Review meaning of factor)

2. Do application A.

3. From application A the students will recognize that some numbers have several pairs of factors, while others have only one pair, namely 1 and itself.

   Define a prime number as a number greater than one which has only 1 and itself as factors.

   Define a composite number as a number which is not prime.

4. Do application B.

5. Define common factor and greatest common factor (GCF). What are the factors of 28 and 42? List as follows:

   28 factors 1,2,4,7,14,28

   42 factors 1,2,3,6,7,14,21,42

   Observe that common factors are 1,2,7,14 and the GCF is 14.

6. Do application C.

7. Develop a procedure for reducing fractions.

   Consider: $\frac{28}{42}$. Students know GCF = 14

   Express: $\frac{28}{42} = \frac{14 \times 2}{14 \times 3} = \frac{2}{3}$

8. Do application D

APPLICATIONS:

A. Express each of the following numbers as a product of two factors in one or more ways:

1. 12	4. 17
2. 20	5. 8
3. 25	6. 11

B. List the factors of each of the following numbers and tell whether they are prime or composite.

			Prime or composite
1.	12	factors 1,2,3,4,6,12	
2.	10		
3.	9		
4.	16		
5.	48		
6.	47		
7.	39		
8.	2		

C. Using the results in application B, determine the GCF of each pair of numbers.

1. 10 and 12
2. 12 and 16
3. 16 and 48
4. 2 and 9
5. 16 and 39
6. 9 and 39

D. Reduce each of the following to lowest terms using the GCF.

1. $\frac{10}{12}$
2. $\frac{12}{16}$
3. $\frac{16}{48}$
4. $\frac{2}{9}$
5. $\frac{16}{39}$
6. $\frac{9}{39}$
7. $\frac{13}{52}$
8. $\frac{12}{52}$
9. $\frac{30}{100}$
10. $\frac{15}{35}$

SUMMARY:

1. Review procedure for reducing fractions to lowest terms.
2. How can you determine whether or not a fraction is in the lowest terms? (When the GCF of the numerator and denominator is 1.)

LESSON 9

AIM: To introduce the concept of a sample space

PERFORMANCE OBJECTIVES: The student will be able to:

- list all possible outcomes of a probability experiment (the sample space)
- calculate the probability of a given outcome in an experiment after listing the sample space

VOCABULARY:

New Terms: sample space, die, dice

CHALLENGE PROBLEM:

Suppose you had a choice of four shirts - one green, one blue, and two tan. You also had a choice of three pairs of jeans - one blue, one green, and one tan. How many different outfits could be made if color combinations were unimportant?

DEVELOPMENTS:

1. Discuss challenge problem. Have students recognize that it is easy to solve this problem if the data is arranged in a table as follows:

SHIRT

JEANS	green (G)	blue (B)	tan (T_1)	tan (T_2)
blue (b)	bG	bB	bT_1	bT_2
green (g)	gG	gB	gT_1	gT_2
tan (t)	tG	tB	tT_1	tT_2

In this table bG represents blue jeans and a green shirt.

(Note: Indicate that since there are two tan shirts, each must be listed separately and marked as T_1 and T_2.)

2. Define Sample Space as a list or table of all possible outcomes.
3. Have students answer the following questions based on the given sample space. Which outfits will have:
   a. both a tan shirt and tan jeans? tT_1, tT_2
   b. a green shirt and tan jeans? tG
   c. shirt and jeans the same color? gG, bB, tT_1, tT_2
4. Use the responses in item 3 to determine the probability that particular outfits will occur when a shirt and a pair of jeans are selected at random.

   P (green shirt, tan jeans) = $\frac{1}{12}$

   Review definition of probability of success.
5. Do applications.

APPLICATIONS:

1. List the sample space for choosing one dinner and one dessert from the menu.

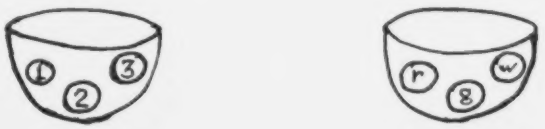
BIG MIX'S

M E N U

DINNERS:	BURGERS
	CHICKEN
	RIBS
DESSERTS:	CAKE
	PUDDING

2. Two bowls contain marbles as shown in the drawing.

BOWL 1 BOWL 2



One marble is drawn from each bowl at random.

Complete the table showing the sample space.

		BOWL 1		
		1	2	3
Bowl 2	r	1, r		
	g	1, g		
	w			

3. Show the sample space for the experiment "rolling one red die and one white die."

4. The teacher should provide students with a variety of problems concerning the probability of various events, using the sample spaces in question 1 - 3. For example: If selection is random, what is the probability of

   a. ribs and cake

   b. chicken or burgers, and cake

   c. fish and pie

   d: 3,g

   e: r and a number

SUMMARY:

Review the meaning of sample space, a procedure for listing the sample space, and the rule for determining the probability of an event given the sample space.

LESSON 10

AIM: To use sample spaces to predict the outcomes of experiments

PERFORMANCE OBJECTIVES: The student will be able to:

- describe a sample space and determine the probability of an event before performing an experiment.

VOCABULARY:

New Term: divisibility

CHALLENGE PROBLEM:

If a coin and a die are tossed, find P (H,1).

DEVELOPMENT:

Continue finding probabilities using sample spaces. The following problems will supply a variety of examples where the sample space listing is essential in determining probability ratios.

APPLICATIONS:

A. A coin and a six-sided die are thrown. Find the following probability ratios:

1. P (H,1)
2. P (H,2)
3. P (H,3)
4. P (H,4)
5. P (H,5)
6. P (H,5)
7. P (T,1)
8. P (T,2)
9. P (T,3)
10. P (T,4)
11. P (T,5)
12. P (T,6)
13. P (H, prime number)
14. P (T, prime number)
15. P (H, composite number)
16. P (H, number divisible by 2)
17. P (T, number divisible by 3)
18. P (H, number divisible by 6)

(Note: review meaning of divisibility)

B. Mary wanted to rearrange the letters in the word "tar" to form other three-lettered arrangements.

1. How many different ways could she rearrange the letters?
2. Find the probability of the arrangement which begins with an a, a t, a r, a vowel, a consonant.

C. A team has four pitchers (Jose, David, Louis, Charlie) and three catchers (Gary, Brian, Stephen).

1. How many different ways can the coach select the pitcher and catcher?
2. Find the probability that Jose will pitch and Brian will catch.
3. Find the probability that David will not play.

D. There are three pairs of different colored socks in a drawer (red, blue, black).

1. List the sample space representing all possible ways of removing two socks at a time.
2. Find the probability of removing a pair of socks of the same color; of different colors.

CLIFFHANGER:

If the $P(H) = \frac{1}{2}$ when tossing coin, how many heads will appear when a coin is tossed 100 times?

LESSON 11

AIM: To determine the expected number of outcomes using the probability ratio

PERFORMANCE OBJECTIVES: The student will be able to:

- calculate the predicted number of outcomes by using the rule. Predicted number of successful outcomes = probability of success times the number of trials

VOCABULARY:

New Terms: predictable, expected number

CHALLENGE PROBLEM:

If a coin and a die are tossed, find P (H,1).

DEVELOPMENT:

1. The cliff-hanger problem from the previous lesson should be used to motivate the lesson. Elicit the concept of P (H) = $\frac{1}{2}$ can be used to determine the predicted number of outcomes when tossing the coin 100 times. Perform this experiment in class and compare both results.

2. Discuss the results of this experiment. Were these results what was expected or predicted? Data among the groups can be collected and used to show that the experimental data approximates the predicted outcome: $\frac{1}{2}$ of 100 or 50 times, but results of individual experiments can differ from the predicted number of successes. Use a table format to generalize the findings. For example:

OUTCOME	PROBABILITY	PREDICTED OUTCOMES
H	$\frac{1}{2}$	$\frac{1}{2}$ of 100 = $\frac{1}{2} \times 100 = 50$
T	$\frac{1}{2}$	$\frac{1}{2}$ of 100 = $\frac{1}{2} \times 100 = 50$

3. Do similar experiments for predicting total number of outcomes using two coins, three coins, a spinner, etc. Elicit the generalization that:

   P (event) times number of trials = predicted number or occurences of event.

4. Do applications.

APPLICATIONS:

1. Multiply:

   a. $\frac{3}{4} \times 20$

   b. $\frac{5}{8} \times 32$

   c. $\frac{2}{3} \times 21$

   d. $\frac{1}{4} \times 15$

   e. $\frac{4}{9} \times 18$

   f. $\frac{3}{8} \times 24$

   g. $\frac{4}{7} \times 28$

   h. $\frac{2}{5} \times 12$

   i. $\frac{1}{3} \times 9$

   j. $\frac{7}{9} \times 12$

2. If a multiple-choice test has 20 questions with four choices for each question, how many questions should students expect to get correct if they guess randomly on each question?

   Solution: $P(s) = \frac{1}{4}$ Number correct = $\frac{1}{4} \times 20$

3. A spinner with sectors A, B and C of equal area is spun 42 times. How many times do you expect it to stop on A?

SUMMARY:

1. Review the definition and purpose of probability ratios.
2. Review the definition and purpose of sample space.

SUGGESTED TEST ITEMS

1. Arrange the following set of scores in a frequency table:

   90, 80, 70, 85, 70, 60, 90, 85, 65, 60, 80, 85

FREQUENCY TABLE

SCORE	TALLY	FREQUENCY
90		
85		
80		
75		
70		
65		
60		

2. Use the frequency table given below to answer each of the following:

   a. What is the mode?

   b. What is the median salary?

   c. Which of these two measures of central tendency is more descriptive of ABC Employees' salaries? Explain.

ABC EMPLOYEES' SALARIES

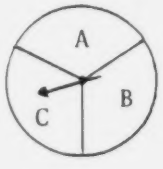
ANNUAL SALARY	FREQUENCY
$12,000	5
$15,000	3
$20,000	4
$25,000	2
$50.000	1

*3. Find the mean of 52, 80, 80, 100

*4. In the last five years, a baseball team won 101, 89, 100, 97, and 83 games. What is the median number of games won?

*5. The ages of four friends are as follows: 14, 14, 16, 17. What is the median age?

*6. A spinner has 3 equal areas labeled A, B, and C. What is the probability that the spinner will stop in area A on the next spin?



*7. Without looking, Mary picked one marble out of a bag containing 1 white, 2 red, 1 blue, and 3 green marbles. What is the probability that she picked a red marble?

*8. Write $\frac{12}{15}$ as a fraction in lowest terms.

*9. What is $\frac{5}{8}$ of 40?

10. An experiment consists of tossing a coin and picking one chip from a bag with chips numbered 1, 2, and 3. One possible outcome is H,1. List the rest of the sample space.

11. The probability of winning in a particular game is $\frac{2}{15}$. What is the probability of losing?

12. If the probability of success in one trial is $\frac{2}{3}$, how many successes should be expected in 60 trials?

*13. Expressed as a fraction in lowest terms, $\frac{45}{100}$ is equal to

a) $\frac{4}{5}$ b) $\frac{5}{4}$ c) $\frac{18}{49}$ d) $\frac{9}{20}$

*14. Which is a prime number?

a) 15 b) 16 c) 17 d) 18

*15. What is the greatest common factor of 6 and 12?

a) 6 b) 2 c) 3 d) 12

*16. What is the probability of drawing a heart from a standard deck of 52 playing cards?

a) $\frac{1}{52}$ b) $\frac{1}{2}$ c) $\frac{1}{3}$ d) $\frac{1}{4}$

*17. Which is a prime factor of 25?

a) 50 b) 2 c) 5 d) $12\frac{1}{2}$

Model Mid-Year Examination

1. Add:
$$\begin{array}{r} 5{,}127 \\ 928 \\ +\ 561 \\ \hline \end{array}$$

2. Subtract 293 from 3,127

3. Multiply:
$$\begin{array}{r} 602 \\ \times\ 86 \\ \hline \end{array}$$

4. Find the remainder in the problem:
$$21\overline{)\ 4{,}567}$$

5. Multiply:
$$\begin{array}{r} 6.28 \\ \times\ .8 \\ \hline \end{array}$$

6. Add: $\frac{2}{5} + \frac{1}{3}$

7. Subtract:
$$\begin{array}{r} 8\frac{5}{9} \\ 3\frac{7}{9} \\ \hline \end{array}$$

8. Multiply: $\frac{4}{9} \times \frac{5}{11}$

9. Add: $7.75 + 4.5$

10. Round 24.75 to the nearest whole number.

11. Divide $5\overline{)\ 72.5}$

12. From a ten dollar bill, how much change would be received for a purchase of $6.09?

13. Add: +7 and -9.

14. Round off $2.9849 to the nearest cent.

15. Find the diameter of the circle if the radius is 10 cm.

16. Write the symbol($<$, =, $>$) which makes the following statement true.

$$\frac{7}{12} \square \frac{1}{2}$$

17. Find the perimeter of a rectangular wall measuring 10.2 meters by 13.3 meters.

18. What is the average (mean) grade of the following test marks: 75, 85, 80, 60?

19. Write the numeral for five thousand, two hundred twenty and three tenths.

20. What is the value of $\sqrt{144}$?

21. Using the formula $C = 2\pi r$, what is the circumference of a circle whose radius is 10.? (Use $\pi = 3.14$)

a) 6.28 b) 31.4 c) 62.8 d) 314

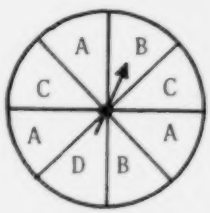
22. Which is equal to $\frac{14}{3}$?

a) $3\frac{3}{4}$ b) $4\frac{1}{3}$ c) $4\frac{1}{2}$ d) $4\frac{2}{3}$

23. Which has the greatest value?

a) -8 b) +2 c) 0 d) -5

24. A spinner has 8 equal areas labeled A, B, C, or D. What is the probability that the spinner will stop in area A on the next spin?



a) $\frac{1}{8}$ b) $\frac{1}{4}$ c) $\frac{3}{8}$ d) $\frac{3}{5}$

25. Which value of x makes the statement $7x < 38$ true?

a) 5 b) 6 c) 7 d) 8

26. Which is equal to $1\frac{3}{4}$?

a) 0.75 b) 1.75 c) 1.25 d) 1.34

27. In the morning the temperature was $-5^{o}C$. In the afternoon the temperature was $4^{o}C$. How much did the temperature change?

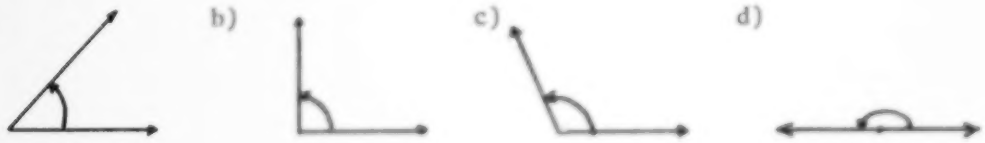
a) $+1^{o}C$ b) $-1^{o}C$ c) $+9^{o}C$ d) $-9^{o}C$

28. The additive inverse of -3 is:

a) 1 b) $\frac{1}{3}$ c) 3 d) $-\frac{1}{3}$

29. Which angle best approximates an angle of 120^{o}?

a) b) c) d)



30. The perimeter of a square is 32. The length of one of its sides is:

a) 8 b) 16 c) 32 d) 128

31. How many centimeters are in 8 meters?

a) 8 b) 80 c) 800 d) 8000

32. In a deck of 52 playing cards there are 13 hearts. If one card is drawn at random what is the probability that it is a heart.

a) $\frac{1}{52}$ b) $\frac{1}{13}$ c) $\frac{1}{4}$ d) 13

33. During a seven day period a pizza shop sold 68, 70, 70, 73, 83, 86, and 89 pizza pies. What is the median number of pies sold?

a) 70 b) 73 c) 77 d) 78

34. In the numeral 42,013 which digit is in the ten thousands place?

a) 1 b) 2 c) 3 d) 4

35. Which of the following numbers has only two factors?

a) 7 b) 8 c) 9 d) 10

36. A triangle has angles of 36° and 74°. How many degrees are in its third angle?

a) 70° b) 80° c) 110° d) 180°

37. 3^4 is equal to:

a) 12 b) 64 c) 81 d) 4000

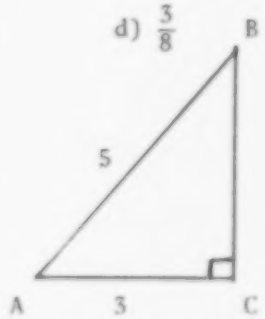
38. A plane left at 10:45 a.m. and landed $2\frac{1}{2}$ hours later. What time did it land?

a) 12:15 p.m. b) 12:45 p.m. c) 1:15 p.m. d) 1:45 p.m.

39. Which fraction has the smallest value?

a) $\frac{3}{4}$ b) $\frac{3}{5}$ c) $\frac{3}{7}$ d) $\frac{3}{8}$

40. In the diagram of right triangle ABC, AB = 5 and AC = 3. Using the Pythagorean relationship, what is the length of $\overline{BC}$?



a) 1 b) 2 c) 3 d)4

41. Debra bought a stereo on the installment plan. She paid $50 down and made 8 monthly installments of $22 each. What was the total cost of the stereo?

a) $166 b)$176 c) $226 d)422

42. What is the least common denominator of $\frac{1}{3}$, $\frac{3}{8}$, and $\frac{5}{16}$?

a) 15 b) 16 c) 24 d) 48

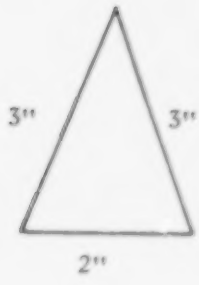
43. The triangle shown is called:

a) equilateral

b) isosceles

c) right

d) scalene



44. Find the mode of the following contributions collected by one student in a walk-a-thon:

$2.50, $2.50, $2.50 $2.50, $3.50, $3.50, $4.50, $4.50, $7.50

a) $2.50 b) $3.50 c) 4.50 d)$7.50

45. Mr. Washington is paid each week according to the following schedule:

$6 per hour for the first 30 hours

$9 per hour for each hour after 30 hours

How much does he earn in a week where he works 41 hours?

a) $179 b)$ 246 c) $279 d) $369

46. Add: (+9) + (-8) + (-1)

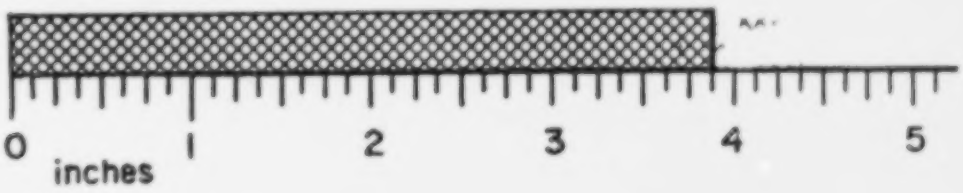
a) 0 b) -2 c) -18 d) +18

47. The probability of an event happening is $\frac{3}{5}$.

What is tne probability the event will not happen?

a) 0 b) $\frac{2}{5}$ c) $\frac{3}{5}$ d) $\frac{5}{3}$

48. In the diagram below, what is the length of the bar?



a) 3.7 in. b) $3\frac{7}{8}$ in. c) $3\frac{15}{16}$ in. d) $4\frac{1}{8}$

49. Which phrase can represent "three less than twice a number?"

a) 3 - 2n b) 2n - 3 c) 2 - 3n d) (2-3)n

50. The cost of a long-distance telephone call is given by the following:

$1.05 for the first two minutes

.45 for each additional minute

What is the total cost for a 6-minute long-distance call?

a) $2.70 b) $2.85 c) $3.15 d) $6.30

QUESTIONNAIRE - Fundamentals of Mathematics, Part I (1981 edition)

This is an experimental bulletin in mathematics. It needs your suggestions and support before it can be revised for maximum implementation.

1. How many classes in your school pursued this course of study?_____

2. What was the grade level of most of the students who were programmed for this course?_____

3. How helpful do you think this course of study will be in helping the students pass the Regents Competency Examination in mathematics?

_____

_____

_____

4. Which unit or lesson stands out as having been most successful in your classrooms?_____

_____

5. What specific suggestions do you have for the improvement of this bulletin? (Wherever possible, refer to specific pages.)_____

_____

_____

6. What additional comments would you make?_____

_____

_____

This questionnaire was filled out by _____

Title:_____ School_____

Please return by February 16, 1982 to:

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